

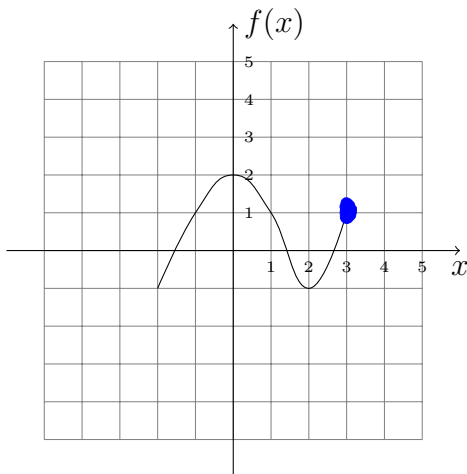
UC Berkeley
Department of Mathematics
Math 32– Midterm 1 Jeff Hicks

Name:

UID:

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- Please do not turn over this page until instructed to do so.
 - This exam contains 7 problems, of which we will score 6 problems. Indicate to us which 6 problems you would like us to grade by checking the small box at the top of the page. We will only grade 6 problems. Each problem is worth 10 points, for a total score of 60 points on this exam.
 - There are no notes or calculators allowed during the examination.
 - Should you finish during the last 15 minutes of the exam period, please remain seated *until we have collected all of the exams* as other students will still be working.
 - This exam contains 8 pages (including this page.)

1. Let $f(x)$ be the function whose graph is drawn below.



- (a) (1 point) What is $f(3)$?

$$f(3) = 1$$

- (b) (2 points) Explain whether or not the function $f(x)$ is one-to-one.

It is not as

$$f(1) = f(-1) = 1.$$

- (a) (2 points) Describe the domain and range of f in interval form.

Domain is $[-2, 3]$

Range is $[-1, 2]$.

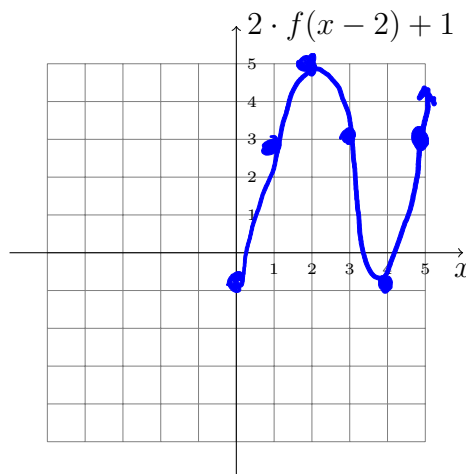
- (c) (5 points) Draw the graph of the function $2 \cdot f(x - 2) + 1$ in the space below.

$$2 \cdot f(x - 2) + 1$$

Shifted right 2

Scaled vert. 2

Shifted up 1



2. Consider the functions given by the tables below.

x	f(x)
1	1
2	3
3	4
4	5
5	2

x	g(x)
1	4
2	4
3	2
4	2
5	3

(a) (1 point) What is $f(3)$?

$$f(3) = 4$$

(b) (2 points) Compute $(f \cdot g)(5)$.

$$f(5) = 2 \quad g(5) = 3 \quad (f \cdot g)(5) = 10$$

(c) (3 points) Compute $(g \circ f)(3)$.

$$g(f(3)) = g(4) = 2$$

(d) (4 points) Compute $(f^{-1} \circ g)(2)$.

$$f^{-1}(g(2)) = f^{-1}(4) = 3$$

3. (a) (5 points) Give the equation of a line containing the points (4, 2) and (2, 1).

$$m = \frac{2-1}{4-2} = \frac{1}{2}$$
$$(y-2) = \frac{1}{2}(x-4)$$
$$y-2 = \frac{1}{2}x-2$$

$y = \frac{1}{2}x$

- (b) (3 points) Find the equation of a line which passes through the origin and is perpendicular to the line from Part (a).

Slope of perpendicular is -2

$y = -2x$

- (c) (2 points) Find the point of intersection between the lines from Parts (a) and (b).

$$\left. \begin{array}{l} y = \frac{1}{2}x \\ y = 2x \end{array} \right\} 2x = \frac{1}{2}x \Rightarrow \frac{3}{2}x = 0 \Rightarrow x = 0$$

$$x=0, y=2x \Rightarrow y=0$$

$(0, 0)$

4. (10 points) Describe the set of real numbers satisfying the inequality $|2x - 2| < x$ using interval notation.

$$|2x - 2| < x$$

$$\text{I} \quad 2x - 2 < 0$$

$$(\Rightarrow x < 1)$$

$$-(2x - 2) < x$$

$$-2x + 2 < x$$

$$2 < 3x$$

$$\frac{2}{3} < x$$

$$\text{So } \frac{2}{3} < x \quad \text{and} \quad x < 1$$

$$\frac{2}{3} < x < 1$$

$$\text{II} \quad 2x - 2 \geq 0$$

$$(\Rightarrow x \geq 1)$$

$$2x - 2 < x$$

$$x - 2 < 0$$

$$x < 2$$

$$\text{So } 1 \leq x < 2$$

$$\frac{2}{3} < x < 1$$

$$1 \leq x < 2$$

$$\frac{2}{3} < x < 2.$$

5. Consider the function

$$f(x) = \frac{x+1}{x-1}$$

(a) (2 points) Write the domain of $f(x)$ using interval notation.

x cannot be 1.

$$(-\infty, 1) \cup (1, \infty)$$

(b) (5 points) Write an equation for the function $f^{-1}(y)$.

$$y = \frac{x+1}{x-1}$$

$$y(x-1) = x+1$$

$$yx - y = x+1$$

$$yx - x = y+1$$

$$x(y-1) = y+1$$

$$x = \frac{y+1}{y-1}$$

$$f^{-1}(y) = \frac{y+1}{y-1}$$

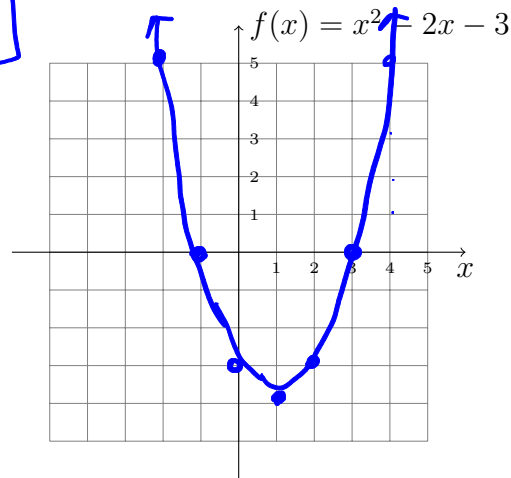
(c) (3 points) Write down the domain and range of $f^{-1}(y)$ using interval notation.

$$\begin{aligned} \text{Range of } f^{-1}(y) &= \text{Domain of } f(x) \\ &= (-\infty, 1) \cup (1, \infty). \end{aligned}$$

$$\text{Domain of } f^{-1}(y) \text{ is } (-\infty, 1) \cup (1, \infty).$$

6. (10 points) Graph the parabola given by the function. $f(x) = x^2 - 2x - 3$ in the space below. Find and mark the zeros and the vertex of the parabola.

by completing the square



$$f(x) = x^2 - 2x + (1 - 1) - 3$$

$$= (x - 1)^2 - 4$$

vertex is (1, -4)

$$f(x) = x^2 - 2x - 3$$

$$= (x - 3)(x + 1)$$

zeros are 3, -1.

7. (10 points) Find all points where the line $y = \frac{3}{4}x$ intersects the circle of radius 5 centered at the origin.

$$y = \frac{3}{4}x$$

$$x^2 + y^2 = 5^2$$

$$x^2 + \left(\frac{3}{4}x\right)^2 = 5^2$$

$$x^2 + \frac{9}{16}x^2 = 25$$

$$16x^2 + 9x^2 = 25 \cdot 16$$

$$25x^2 = 25 \cdot 16$$

$$x^2 = 16$$

$$x = \pm 4$$

$$y = \pm 3$$

(4,3) and
(-4,-3).