Working through an inequality problem

Things to Remember

ⓐ $|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0. \end{cases}$

ⓑ If $a < b$ then

$ax < bx$ if $x > 0$

$ax > bx$ if $x < 0$.

This means if we don't know x, solutions will have to be broken down into multiple cases.

Examples. 1 Find all x so that

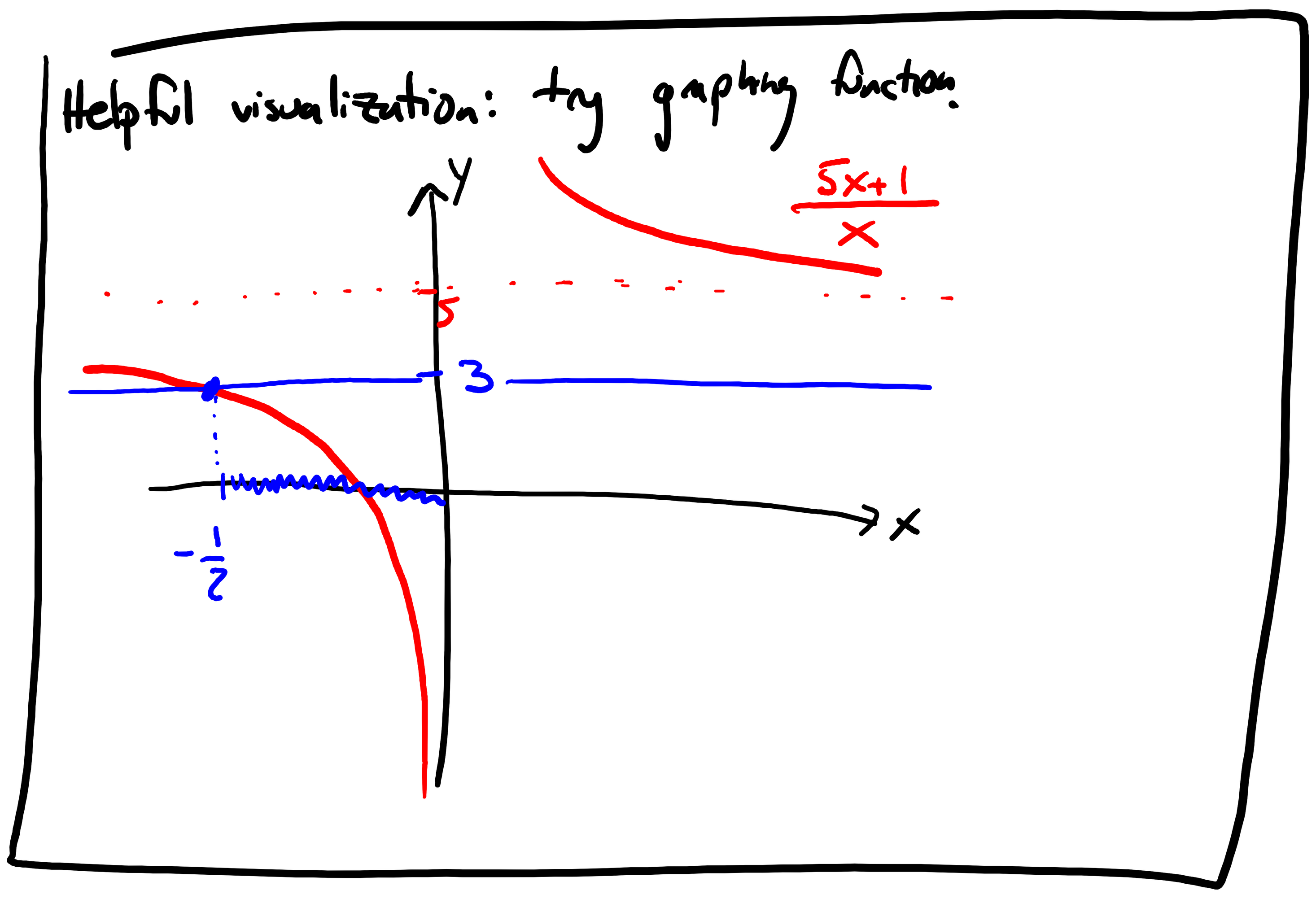
$$\frac{5x+1}{x} < 3$$

First multiply both sids by x. But we don't know if $x > 0$ or $x < 0$. Get 2 cases as a result.

Case 1: $x > 0$ or Case 2: $x < 0$

Case 1: $x > 0$	Case 2: $x < 0$
$5x+1 < 3x$	$5x+1 > 3x$
$2x+1 < 0$	$2x+1 > 0$
$x < -\frac{1}{2}$	$x > -\frac{1}{2}$

Don't Forget that each case had assumptions!



$x > 0$ and $x < -\frac{1}{2}$	$x > -\frac{1}{2}$ and $x < 0$
This case doesn't happen	This case occurs

$\boxed{X \text{ is in interval } \left(-\tfrac{1}{2}, 0\right).}$

Example 2

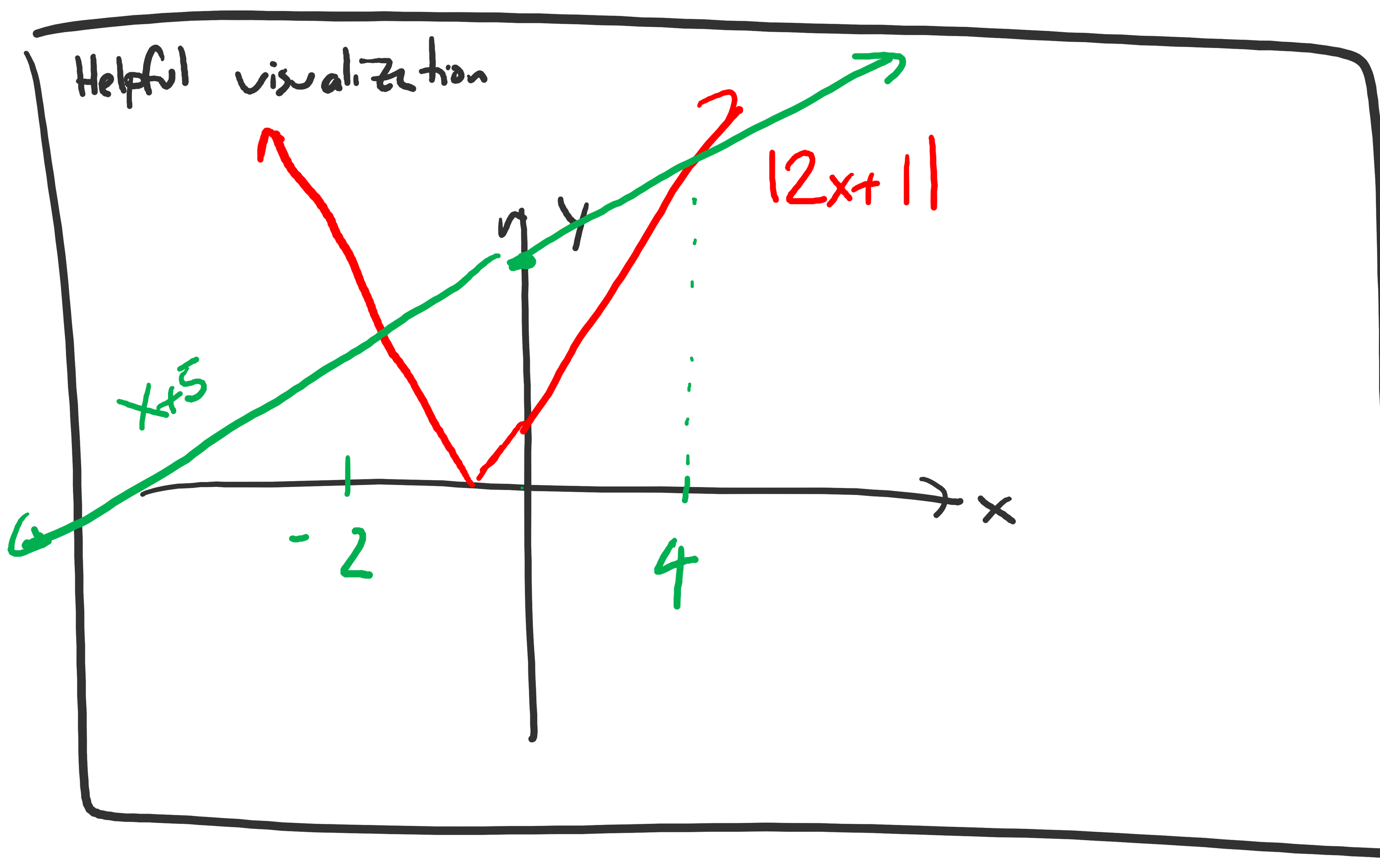
$$|2x+1| \geq x+5$$

We don't know if $2x+1 > 0$ or $2x+1 < 0$?

case 1 $2x+1 \geq 0$ or Case 2: $2x+1 \leq 0$

case 1 $2x+1 \geq 0$	Case 2: $2x+1 \leq 0$
$2x+1 \geq x+5$	$-(2x+1) \geq x+5$
$x \geq 4$	$-2x-1 \geq x+5$
	$-6 \geq 3x$
	$x \leq -2$

Don't forget initial assumption.



	or	
$x \geq 4$, $x \geq -\frac{1}{2}$		$x \leq -2$, $x < -\frac{1}{2}$

X is in $(-\infty, -2) \cup (4, \infty)$.