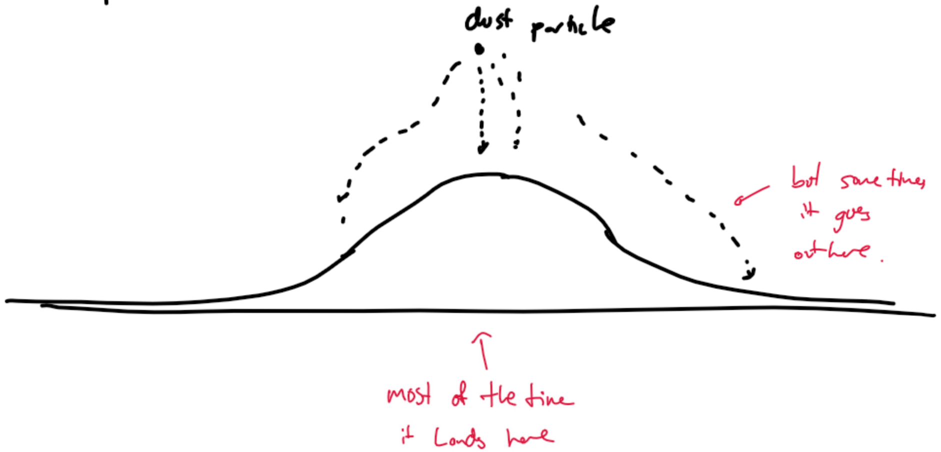
What was our path this semester?

- General theory of Functions & Graphs
  - → Polynomials and their properties
  - → Exponentials & Logs
  - → Sine, Cos & Tan.

Why study these particular functions?

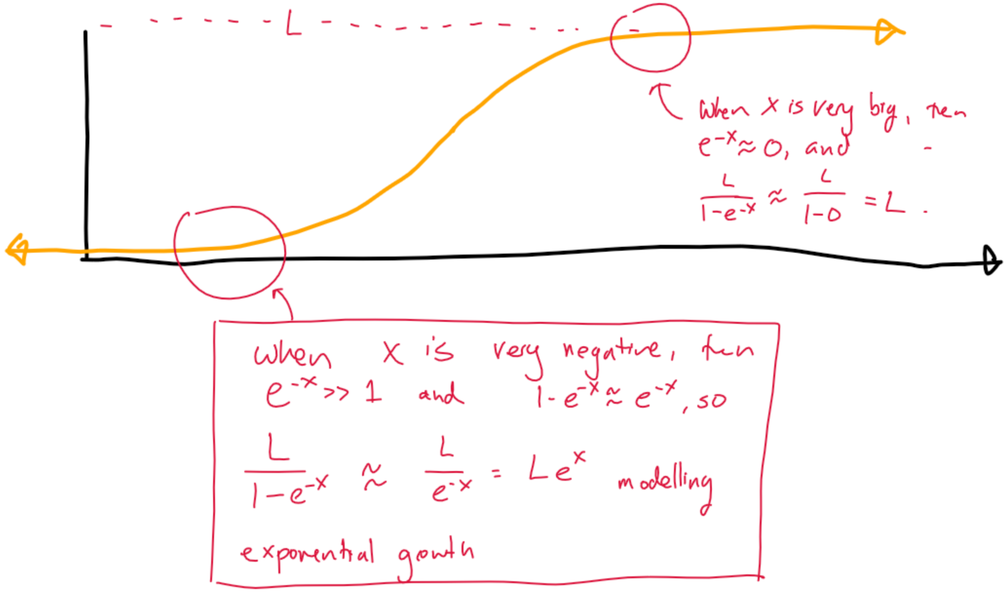
~ They can be used as building blocks for many new functions.

Example: The normal distribution e^{-x^2}

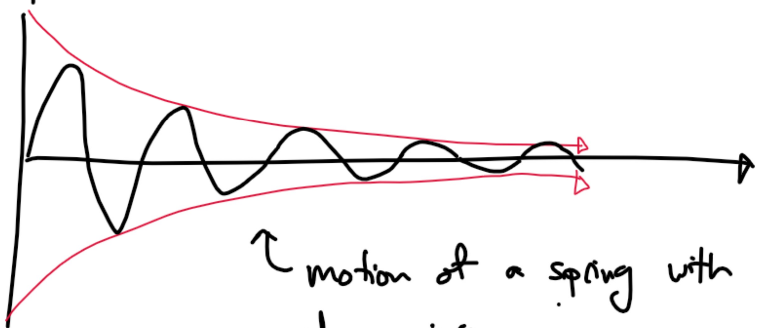


Example: Logistic Growth model

$$p(t) = \frac{L}{1 - e^{-x}}$$



Example: $e^{-t} \sin t$

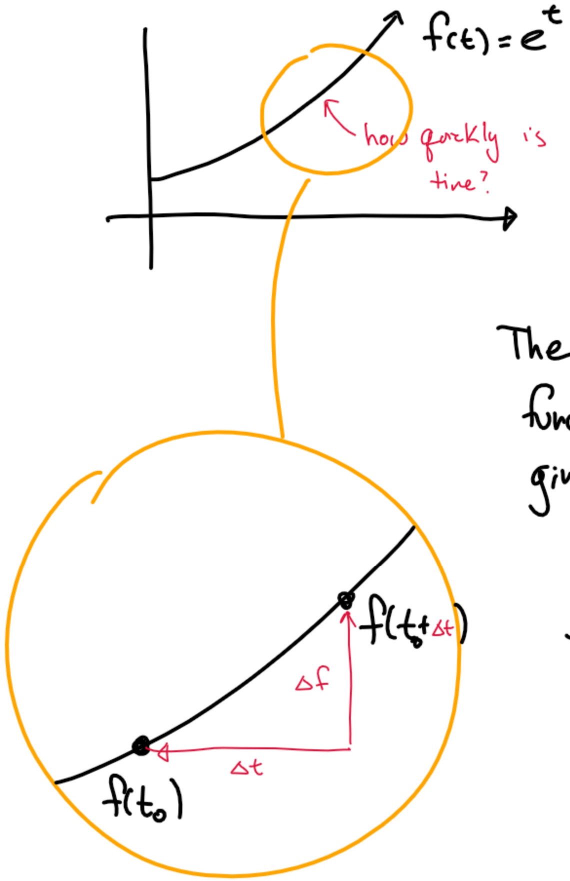


↑ motion of a spring with dampening.

What is Next?

→ Calculus is a variety of mathematical tools which ascribe new properties to old functions.

Example: The rate of change of a function $f(t)$ at a fixed time t.



The change of the function $f(t)$ is given by

$$\frac{\text{change of } f}{\text{change of } t} = \frac{\Delta f}{\Delta t}.$$

Examples: For $f(t) = e^t$

Change of f at time t_0 is

$$\frac{f(t_0+\Delta t) - f(t_0)}{\Delta t} = \frac{e^{t_0+\Delta t} - e^{t_0}}{\Delta t}$$

$$= (e^{\Delta t} e^{t_0} - e^{t_0})/\Delta t$$

$$= (e^{t_0}(e^{\Delta t} - 1))/\Delta t$$

$$\approx (e^{t_0}(1+\Delta t - 1))/\Delta t \quad \text{when } \Delta t \approx 0$$

$$= e^{t_0}\Delta t/\Delta t$$

$$= e^{t_0}$$

Conclusion: The rate of change of the function $f(t) = e^t$ at a specific time t_0 is e^{t_0}.

Takeaway: Phenomenon whose rates of change are dependent on their amount are described by exponentials.

Example: Population Growth
Compounding interest.