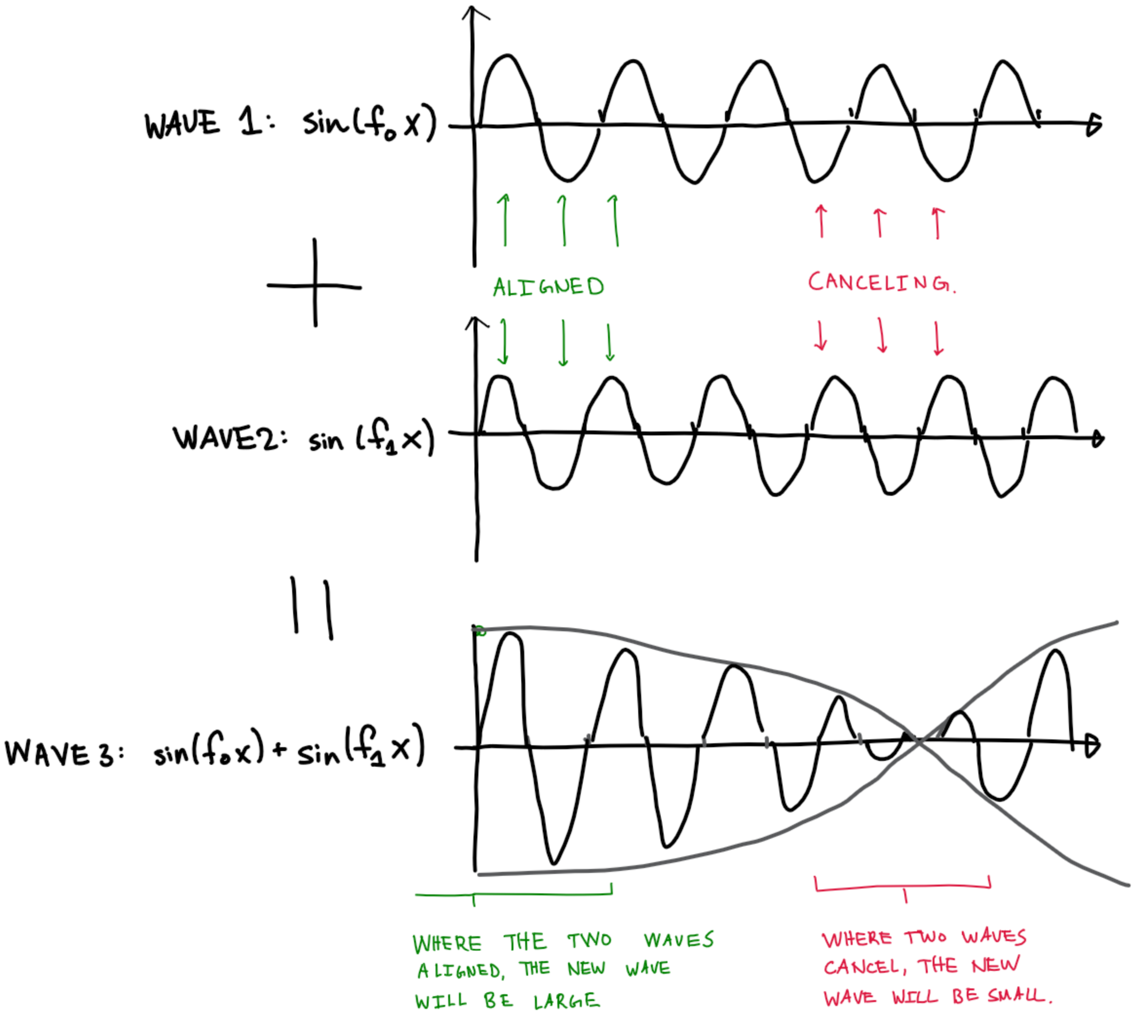
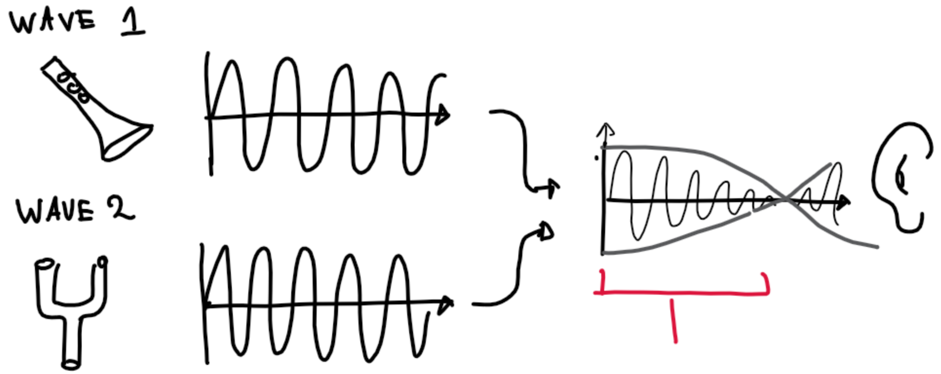
ADDING TOGETHER SINE WAVES



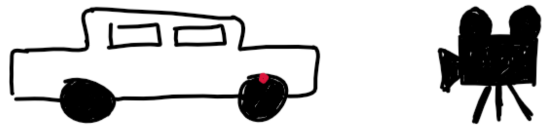
WHEN WE TAKE WAVES OF PERIODS $1/f_0$ AND $1/f_1$ AND ADD THEM TOGETHER, THEY SYNC UP WITH EACHOTHER WITH PERIOD $1/(f_0 - f_1)$.

EXAMPLE: TUNING INSTRUMENTS

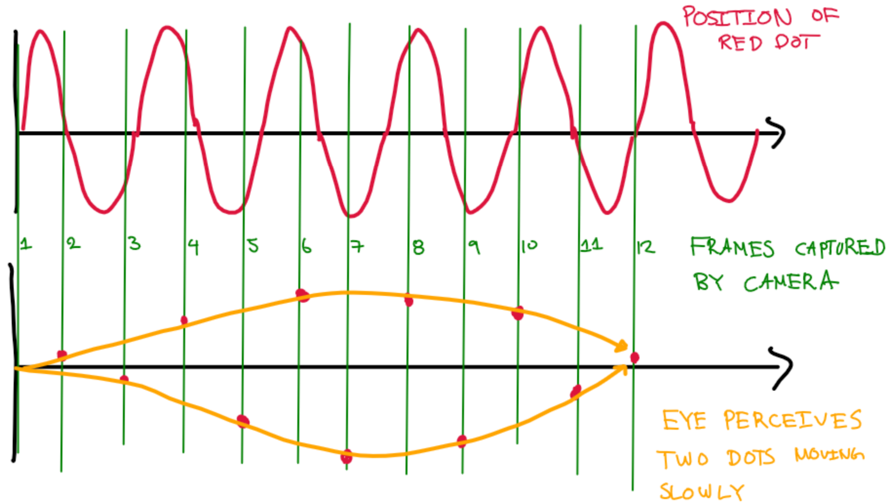


THIS WAVELENGTH MEASURES DIFFERENCE BETWEEN TUNING FORK AND MUSICIAN.

EXAMPLE (IN SPIRIT)



A CAR IS FILMED FOR A MOVIE



MATHEMATICAL UNDERSTANDING

• WE FIRST PROVE A IDENTITY

$$\cos(a+b)\cdot\cos(a-b) = (\cos a\cos b - \sin a\sin b)\cdot(\cos a\cos b + \sin a\sin b)$$

$$= (\cos a\cos b)^2 - (\sin a\sin b)^2$$

$$= \frac{1}{2}((\cos a\cos b)^2 - (\sin a\sin b)) + \frac{1}{2}((\cos a\cos b)^2 - (\sin a\sin b)^2)$$

$$= \frac{1}{2}((\cos a\cos b)^2 + (\cos a\sin b)^2 - (\sin a\sin b)^2 - (\sin a\cos b)^2) + \frac{1}{2}((\cos a\cos b^2) + (\sin a\cos b)^2 - (\sin a\sin b)^2 - (\cos a\sin b)^2)$$

$$= \frac{1}{2}((\cos a)^2 - (\sin a)^2) + \frac{1}{2}((\cos b)^2 - (\sin b)^2)$$

$$= \frac{1}{2}(\cos(2a) + \cos(2b))$$

RESTATING THIS IDENTITY:

$$\cos(u) + \cos(v) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

RETURNING TO OUR EXAMPLE:

WAVE 1 = $\cos(2\pi f_0 t)$

WAVE 2 = $\cos(2\pi f_1 t)$

WAVE 1 + WAVE 2 =

$$2\cos\left(\left(\frac{2\pi(f_0+f_1)}{2}\right)t\right)\cos\left(\left(\frac{2\pi(f_0-f_1)}{2}\right)t\right)$$

WHEN $f_0 \approx f_1 \approx f$ and

$$2\underbrace{\cos(2\pi f t)}_{\text{SOUNDS LIKE OUR ORIGINAL WAVE.}}\cdot\underbrace{\cos(\pi(f_0-f_1)t)}_{\text{NEW "BEAT" FREQ. MODULATING THE AMPLITUDE.}}$$