Half Angle and Angle Sum Identities

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Outline

- Recap: Half angle formula
- Angle sum formula

Half Angle Formulas

Computing $\sin(\theta/2)$

$\cos(2\alpha) = 1 - 2(\sin^2(\alpha))$

Let's compute $\sin(\alpha)$ from $\cos(2\alpha)$

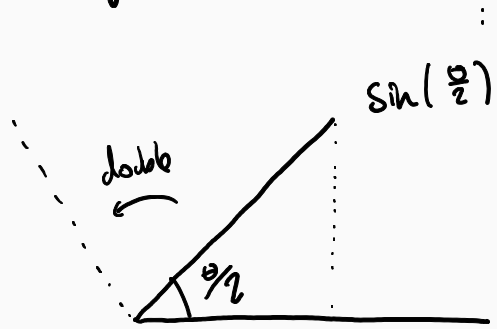
$$\cos(2\alpha) - 1 = -2(\sin(\alpha))^2$$

$$\frac{1-\cos(2\alpha)}{2} = (\sin(\alpha))^2$$

$$\pm\sqrt{\frac{1-\cos(2\alpha)}{2}} = \sin(\alpha)$$

$$\pm\sqrt{\frac{1-\cos(\theta)}{2}} = \sin\left(\frac{\theta}{2}\right)$$

Usually $\alpha = \frac{\theta}{2}$



Identity

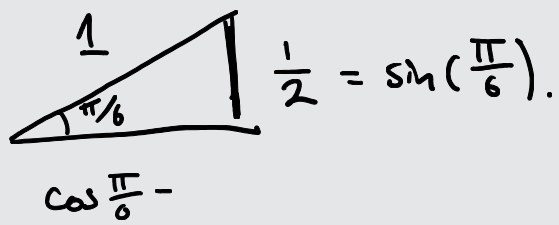
$$\sin(\theta/2) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

Example

Find an exact formula for $\sin(\pi/12)$.

Idea: We know $\sin\left(\frac{\pi}{6}\right)$

Observe that $\frac{\pi}{12} = \frac{1}{2}\cdot\frac{\pi}{6}$



$\frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$$\sin\left(\frac{1}{2}\cdot\frac{\pi}{6}\right) = \pm\sqrt{\frac{1-\cos(\pi/6)}{2}}$$

Computing $\cos(\theta/2)$

$$\cos(2\alpha) = 2(\cos(\alpha))^2 - 1$$

Computing $\tan(\theta/2)$

$$\tan(\alpha) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{2\sin(\theta)\cos(\theta)}{2\cos^2(\theta)}$$

Half Angle Formulas

Identity

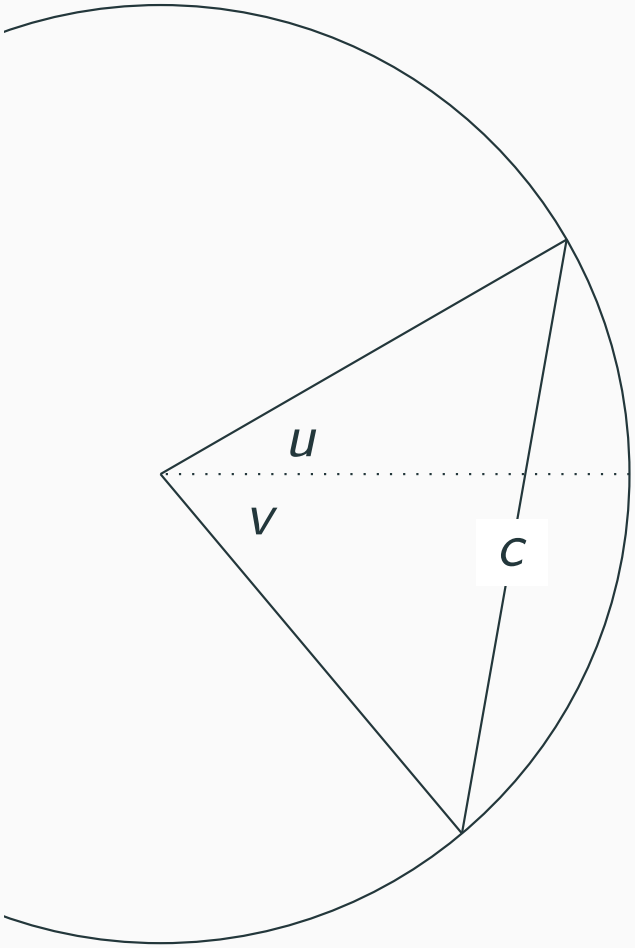
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1+\cos(\theta)}$$

Angle Sum Formula

Computing $\cos(u + v)$.



Angle sum for Cosine

Identity

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v).$

Compute this when $u = 0$.

Application: Right angle identity for cosine

Example: Exactly compute $\cos(5\pi/12)$

Angle sum for Sine

Recall that $\sin(\theta) = \cos(\theta - \pi/2)$

Angle sum for Sine

Identity

$$\sin(u+v) = \cos(u)\sin(v) + \sin(u)\cos(v)$$

Computing $\tan(u+v)$

Angle sum for tangent

Identity

$$\tan(u+v) = \frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}.$$

Exactly compute $\tan(7\pi/12)$

Angle difference formulas

$$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v)$$
$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$
$$\tan(u+v) = \frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}.$$

Application: Beats between two frequencies

Sum of Cosine Waves

$\cos(u+v)\cos(u-v)$

Sound and sin waves.

Temporal Aliasing

From previous slide, we have the identity

$$\cos(u+v)\cos(u-v) = \frac{1}{2}\cos(2u) + \cos(2v)$$