Some Solutions to Homework 1

January 30, 2019

0.1.8

First notice that whenever $\frac{1}{t}$ is rational, we can write

$$\frac{1}{t} = \frac{a}{b}$$

where a and b are integers. This means that we can write

$$t = \frac{a}{b}$$

and therefore t is also written as a fraction with integer numerator and denominator. This is the definition of t being a rational number. In summary,

$$\text{If } 1/t \text{ is rational, } t \text{ is rational} \tag{1}$$

Now suppose that t is irrational, as is stated in the claim. This means that t is not a rational number. Then it cannot be the case that $\frac{1}{t}$ is rational! This is because (1) would imply that t is *rational as well*, which contradicts the starting assumption. Therefore, it cannot be the case that $\frac{1}{t}$ is a rational number when t is an irrational number.

Since $\frac{1}{t}$ cannot be a rational number, this means that $\frac{1}{t}$ is an irrational number.

0.2.10

This is a computation by distribution. We write out every step here, but you have a mnemonic that works well for you, feel free to use it instead.

$$\begin{aligned}(4a-5)^2 =& (4a-5)\cdot(4a-5)\\ =& (4a-5)\cdot 4a - (4a-5)\cdot 5\\ =& 16a^2 - 20a - 20a + 25\\ =& 16a^2 - 40a + 25\end{aligned}$$

0.2.38

Here we find a least common denominator for these fractions, and then simplify.

$$\begin{aligned}\frac{x-3}{5}+\frac{5}{y+2} &= \frac{(x-3)(y+2)}{5(y+2)} - \frac{5\cdot 5}{5(y+2)} \\ &= \frac{x\cdot y-3y+2x-6-25}{5(y+2)} \\ &= \frac{x\cdot y-3y-2x-31}{5(y+2)}.\end{aligned}$$

0.2.60

Compare, for instance:

$$8/(2/2) = 8/1 = 8$$

$$(8/2)/2 = 4/2 = 2$$