Practice Midterm Problems

Name:

0.1. Let S be the bottom 5 faces of the cube with corners at $(\pm 1, \pm 1, \pm 1)$.

Compute the flux of the vector field $\langle x, x^2+y, z^2\rangle$ through the 5 faces of the cube. (Hint: You do not actually have to integrate over the 5 different faces.)

0.2. Consider the vector field $\vec{F}(x,y) = \langle 2x\sin^3(y), 3x^2\sin^2(y)\cos(y)\rangle$. Compute $\int_C \vec{F}\cdot d\vec{r}$ for this vector field along the curve

$$\vec{r}(t) = \langle \cos(t), \sin(t)\rangle$$

as t goes from $-\pi/2$ to $\pi/2$.

0.3. Compute the double integral of the function $\iint_R e^{9x^2+16y^2}\,dA$ over the region R where $9x^2+16y^2 \leq 1$ and $x \geq 0$.

0.4. The boundary of a region is given by the curve $r(t) = \langle t^2, \sin(t)\rangle$ as t goes from $-\pi$ to π. Compute the area of this region.

0.5. Consider the surface S parameterized by

$$\vec{r}(\theta, z) = \langle 1/z^2\cos\theta, 1/z^2\sin\theta, z\rangle$$

where $0 \leq 2\pi \leq \theta$ and $1 \leq z$. This surface looks a bit like a spike going off to infinity along the z axis.

- (Don't Do this!) Compute the surface area of the resulting parameterized surface.
- Consider the vector field
$$\vec{F}(x,y,z) = \frac{1}{2}\langle x, y, 0\rangle.$$
- Compute by hand the flux of the vector field $\vec{F}$ through the surface.
- Let D be disk of unit radius whose center intersects the z axis perpendicularly at $z = 1$. Show the flux of $\vec{F}$ through D is 0.
- Conclude that the volume between S and D is finite.

0.6.

- Draw the region for the integral
$$\int_0^1 \int_x^1 e^{y^2}\,dydx.$$
- Compute the integral (Suggestion: switch order of integration)

0.7. *Bonus, Worth no Points* Show that there is no function from the cube to the square

$$f : [0,1]\times[0,1]\times[0,1] \to [0,1]\times[0,1]$$

so that:

- $f(x,y,0) = (x,y)$
- $f(x,y,t) = (x,y)$ whenever $x, y = 0$ or 1
- $f(x,y,1)$ is completely contained in the boundary of the square.