

QUIZ, APRIL 18TH

NAME:

0.1. **Flux through a curve.** By any means you wish, compute the flux of the vector field

$$\langle x + 2 \sin^3(y), y + 3x^2 \rangle$$

through the square with corners at $(\pm 1, \pm 1)$.

Solution: The divergence of this vector field is 2, so we know (by Green's theorem II) that we can compute the flux by instead integrating the divergence over the bounded region.

$$\int_C F \cdot n \, ds = \iint_R \operatorname{div} F \, dA = \iint 2 \, dA = 2(\text{area}) = 2 \cdot 4 = 8$$

0.2. **Surface Integral I.** Integrate the function $f(x, y, z) = 3z$ over the cone parameterized by

$$\vec{r}(\theta, z) = \langle z \cos \theta, z \sin \theta, z \rangle$$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq z \leq 1.$$

Solution: Setting up this integral, we have that

$$\begin{aligned} dS &= |\vec{r}_\theta \times \vec{r}_z| \, d\theta \, dz \\ &= | \langle -z \sin \theta, z \cos \theta, 0 \rangle \times \langle \cos \theta, \sin \theta, 1 \rangle | \, d\theta \, dz \\ &= z | \langle \cos \theta, \sin \theta, -1 \rangle | \, d\theta \, dz \\ &= z \sqrt{2} \, d\theta \, dz \end{aligned}$$

so

$$\begin{aligned} \int_{z=0}^1 \int_{\theta=0}^{2\pi} f(r(z, \theta)) \, dS &= \int_{z=0}^1 \int_{\theta=0}^{2\pi} 3z \, dS \\ &= 3 \int_{z=0}^1 \int_{\theta=0}^{2\pi} z \cdot z \sqrt{2} \, d\theta \, dz \\ &= 2\pi \sqrt{2} \end{aligned}$$

0.3. **Surface Integral II.** Compute the flux of the vector field $\langle x, y, 0 \rangle$ through the same cone from the first problem. Use any method you wish.

Solution: We need to take the dot product of $\langle x, y, 0 \rangle$ with the $\vec{n}dS = z\langle \cos \theta, \sin \theta, -1 \rangle d\theta dz$. This dot product is $z^2 dz d\theta$. So we integrate

$$\begin{aligned} \int_{z=0}^1 \int_{\theta=0}^{2\pi} F \cdot \vec{n}dS &= \int_{z=0}^1 \int_{\theta=0}^{2\pi} \langle x, y, 0 \rangle \cdot z\langle \cos \theta, \sin \theta, -1 \rangle d\theta dz \\ &= \int_{z=0}^1 \int_{\theta=0}^{2\pi} z^2 d\theta dz \\ &= 2\pi/3 \end{aligned}$$

Bonus Problem, worth no additional Points! Show that there is no surface parameterized by $(\theta, t) \in [0, 2\pi] \times [0, 1]$ such that:

- $\vec{r}(\theta, 0)$ is the equation for unit circle in the xy plane.
- $\vec{r}(\theta, 1)$ is the point $(1, 1, 0)$
- The surface does not intersect the z axis.