Quiz, April 18th

Name:

0.1. **Flux through a curve.** By any means you wish, compute the flux of the vector field

$$\langle x + 2\sin^3(y), y + 3x^2\rangle$$

through the square with corners at $(\pm 1, \pm 1)$.

Solution: The divergence of this vector field is 2, so we know (by Green's theorem II) that we can compute the flux by instead integrating the divergence over the bounded region.

$$\int_C F \cdot n \, ds = \iint_R \text{div} F \, dA = \iint 2 dA = 2(\text{area}) = 2 \cdot 4 = 8$$

0.2. **Surface Integral I.** Integrate the function $f(x, y, z) = 3z$ over the cone parameterized by

$$\begin{aligned}\vec{r}(\theta, z) =& \langle z\cos\theta, z\sin\theta, z\rangle \\ & 0 \leq \theta \leq 2\pi \\ & 0 \leq z \leq 1.\end{aligned}$$

Solution: Setting up this integral, we have that

$$\begin{aligned}dS =& |\vec{r}_\theta \times \vec{r}_z| d\theta dz \\ =& |\langle -z\sin\theta, z\cos\theta, 0\rangle \times \langle \cos\theta, \sin\theta, 1\rangle| d\theta dz \\ =& z|\langle \cos\theta, \sin\theta, -1| d\theta dz \\ =& z\sqrt{2} \;\; d\theta dz\end{aligned}$$

so

$$\begin{aligned}\int_{z=0}^{1} \int_{\theta=0}^{2\pi} f(r(z,\theta)) dS =& \int_{z=0}^{1} \int_{\theta=0}^{2\pi} z \, dS \\ =& \int_{z=0}^{1} \int_{\theta=0}^{2\pi} z \cdot z\sqrt{2} \;\; d\theta dz \\ =& 2\pi\sqrt{2}\end{aligned}$$

0.3. **Surface Integral II.** Compute the flux of the vector field $\langle x, y, 0\rangle$ through the same cone from the first problem. Use any method you wish.

Solution: We need to take the dot product of $\langle x, y, 0\rangle$ with the $\vec{n}dS = z\langle \cos\theta, \sin\theta, -1\rangle d\theta dz$. This dot product is $z^2 dz d\theta$. So we integrate

$$\begin{aligned}\int_{z=0}^{1}\int_{\theta=0}^{2\pi} F\cdot\vec{n}dS =& \int_{z=0}^{1}\int_{\theta=0}^{2\pi}\langle x, y, 0\rangle\cdot z\langle\cos\theta, \sin\theta, -1\rangle d\theta dz\\ =& \int_{z=0}^{1}\int_{\theta=0}^{2\pi} z^2 d\theta dz\\ =& 2\pi/3\end{aligned}$$

Bonus Problem, worth no additional Points! Show that there is no surface parameterized by $(\theta, t) \in [0, 2\pi]\times[0, 1]$ such that:

- $\vec{r}(\theta, 0)$ is the equation for unit circle in the xy plane.
- $\vec{r}(\theta, 1)$ is the point $(1, 1, 0)$
- The surface does not intersect the z axis.