Quiz, March 28th

Name:

Spherical Integration. Set up an integral which computes the volume of the region drawn below:
Solution: The region was the volume bounded below by $x^2+y^2=z^2$, and bounded above by $x^2+y^2+z^2=1$. This means that it is the region between a cone with a point at the origin, and the spere. The θ values that we have in this region go from 0 to 2π. The cone restricts us the thet ϕ values of $0 \leq \phi \leq \frac{\pi}{4}$. This is because the cone meets the plane at a 45 degree angle. Finally, the value of ρ is between 0 and 1. Therefore, the volume of the region is given by

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^1 1\rho \sin(\phi)\ d\rho d\phi d\theta$$

Line Integrals I. Compute the integral of the function $f(x,y) = x+y$ over the curve

$$x(t) = t$$
$$y(t) = t$$

as t goes from 0 to 1.
Solution: Here, we have that

$$ds = \sqrt{(x'(t))^2 + (y'(t))^2}dt = \sqrt{2}dt$$

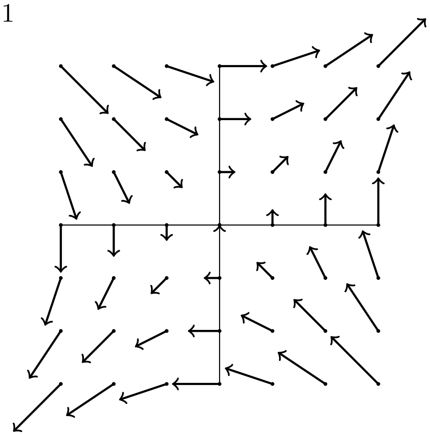
mskinh out integral

$$\int_0^1 f(t,t)\sqrt{2}dt = \int_0^1 2t\sqrt{2}dt = \sqrt{2}$$

Identify the Vector Field.

$$\begin{array}{cc} A & \langle x, y\rangle \\ B & \langle -y, x\rangle \\ C & \langle x-y, x+y\rangle \\ D & \langle y, x\rangle \end{array}$$

1



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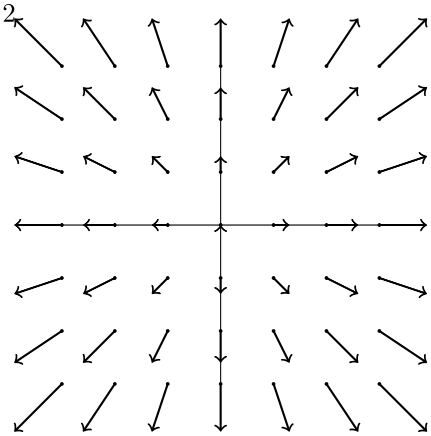

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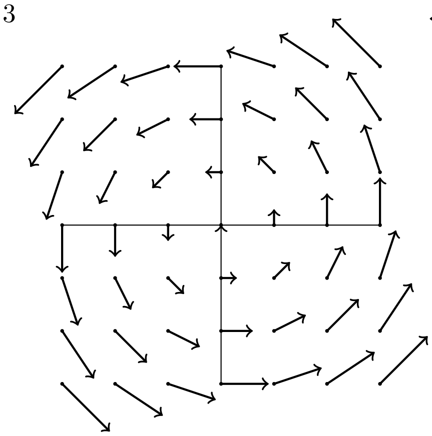

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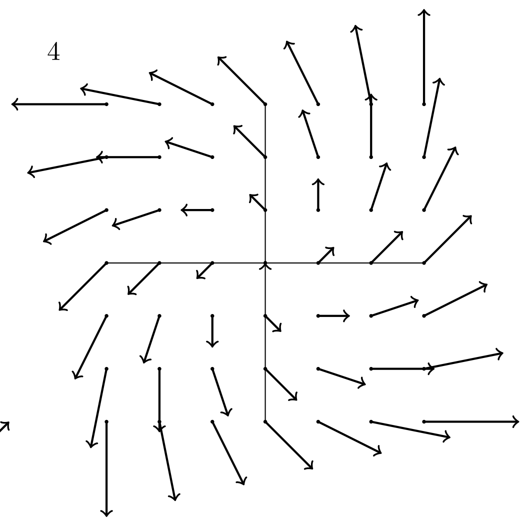

Solution: For this one, notice that $\langle x, y\rangle$ is the only one which has horizontal direction when $y = 0$. Therefore, the second graph (the one pointing all out) is A.

Notice that $\langle x-y, x+y\rangle$ is neither vertical or horizontal along the x axis. So, it must be the 4th vector field.

For vector fields B and D, notice that B must point left when y is positive. This is 3.

That leaves the last vector field d to be number 4.

Bonus Problem. Let $C(t) = (r(t) : \theta(t))$ be a polar parameterized curve, with $r(t) > 0$ and $C(0) = C(1)$. Consider the vector field $\vec{F} = \langle y/(x^2+y^2), -x/(x^2+y^2)\rangle$. Show that

$$\int_C \vec{F} \cdot dr$$

counts the number of times that the curve C winds around the origin.