QUIZ, FEB 29

NAME:

0.1. **Tangent Plane.** Suppose that $\nabla f(2,3) = \langle 3,4\rangle$ and that $f(2,3) = 2$. Find the equation to the plane tangent to the graph of f going which contains the point $(2,3,2)$.
Solution: We have that the equation for the tangent plane to a graph is given by $(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(y - y_0)$. Given that, we have

$$z - 2 = 3(x - 2) + 4(y - 3).$$

0.2. **Minimizing a function.** Find the point on the upper hemisphere

$$f(x,y) = \sqrt{1 - x^2 - y^2}$$

which is closest to the point (2, 3, 6). **Solution:**Let's create a function which computes the distance of a point on the sphere to the point $(2,3,6)$. The distance of a point (x,y,z) from (x_0, y_0, z_0) is given by

$$d(x,y,z) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

Since $z = f(x,y)$, we are looking to minimize $d(x,y,f(x,y))$. This is the same as minimizing $(d(x,y,f(x,y))^2$, so we can try to minimize

$$g(x,y) = (x - x_0)^2 + (y - y_0)^2 + ((\sqrt{1 - x^2 - y^2}) - z_0)^2$$

Plugging the the point we are trying to minimize to, we get

$$(x - 2)^2 + (y - 3)^2 + (\sqrt{1 - x^2 - y^2} - 6)^2$$

Taking the derivative in the x direction, we get after simplification

$$\frac{12x}{\sqrt{1 - x^2 - y^2}} - 4$$

Taking the derivative in the y direction, we get after simplification

$$\frac{12y}{\sqrt{1 - x^2 - y^2}} - 6$$

We then have that

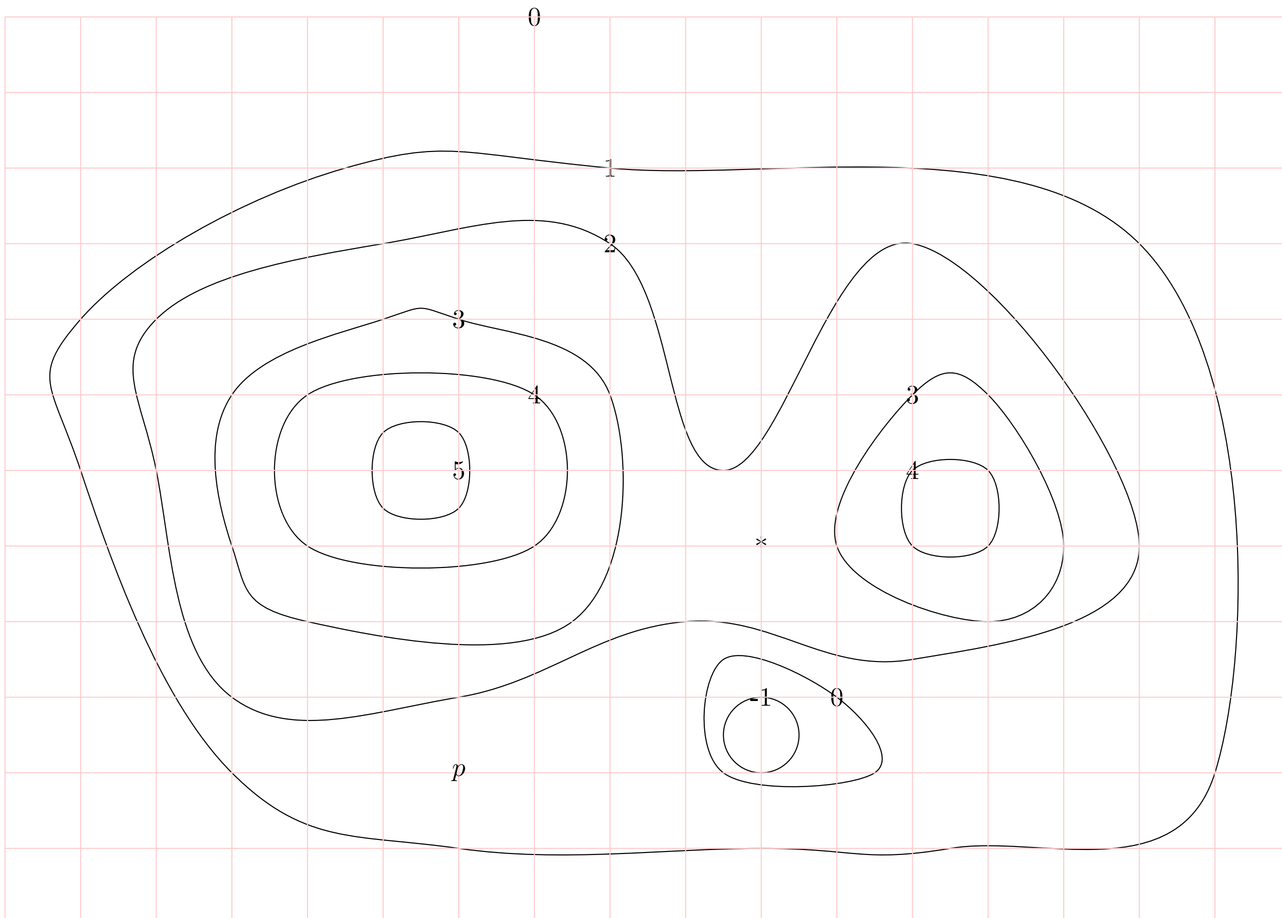
$$(y - 3)/y = (x - 2)/x$$

.
This gives that $x = 2y/3$. Plugging this into the first equation gives

$$\frac{12y}{\sqrt{1 - 2y/3^2 - y^2}} - 6$$

which has solution $y = 3/7$. This means that $x = 2/7$, and $z = 6/7$.

0.3. **Reading Contour Graphs.**



Mark the Local Maximas, Local Minimas, and Saddle points on this contour plot. Then estimate directional directives at p

- In the $\langle 1, 0\rangle$ direction?
- In the $\langle 0, 1\rangle$ direction?
- In the $\langle 0, -1\rangle$ direction?
- In the $\langle 3/\sqrt{13}, 2/\sqrt{13}\rangle$ direction?

Solution:: There is a maximum at the 5, and the 4 on the right. There is a minimum at (-1). There is a saddle point at the location of the $*$.

To compute the partial derivatives, we estimate the value of p to be 1.5. The to take $\partial_y f$, we take a small change in the y direction, say to the contour right above it. This push-off in the y direction is about 1, and the corresponding change in z is to $z = 2$. So, we would estimate

$$\partial_y f \sim \frac{\delta z}{\delta y} = (2 - 1.5)/(1) = .5$$

Similarly, if we go right from the point p, we stay almost the same distance between level curves. We might then estimate the change in the altitude to be zero as we travel in the x-direction from p. Therefore, we would estimate $\partial_x f(p) = 0$. To compute the partial derivative in a different direction, you can take the dot product with that direction (which is a unit vector) with the gradient. We have estimated the gradient to be

$$\nabla f = \langle 0, .5\rangle$$

so our first guess for what the derivative in the direction $\langle 3/\sqrt{13}, 2/\sqrt{13}\rangle$ be $1/\sqrt{13}$.

Bonus Problem. *Worth no points!* There are two roads from Evans to Wheeler , and two people are able to walk from Evans to Wheeler along the two routes while holding a 50 foot long rope. Prove that if one person goes from A to B along one route, and the other from B to A along the other route, then at some point they are at most 50 feet away from one another.