PRACTICE MIDTERM, FEB 29

NAME:

0.1. The line AB is given by $\vec{a}(t) = \langle 2t+1, 3t+1, 2\rangle$ and the line CD is given by $\vec{b}(s) = \langle s, s, s\rangle$.

(a) Find a line ℓ which intersects both the line AB and the line CD at right angles.

(b) Find a function $D(s,t)$ which computes the distance between $\vec{a}(t)$ and $\vec{b}(s)$. Minimize this function to find the smallest distance between the lines AB and CD.

(c) Confirm that the line ℓ minimizes the distance between AB and CD.

0.2. Draw a contour plot for a function which has exactly 4 critical points– 1 saddle, 2 maximums and 1 minimum.

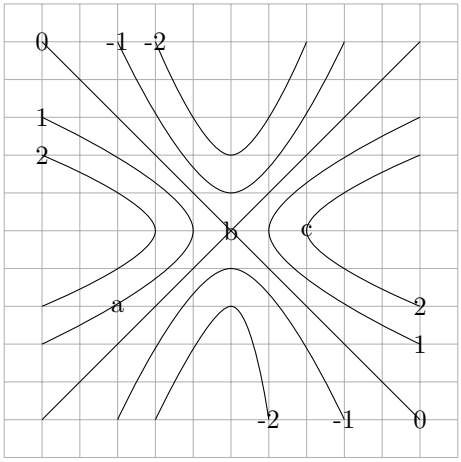
0.3. The upper hemisphere is given by the function $f(x,y) = \sqrt{1-x^2-y^2}$. Using the gradient, compute the tangent plane to this graph at the point $(a, b, \sqrt{1-a^2-b^2})$.

0.4. Let $f(x,y) = x^2 + y^2$. Suppose that we know that $\vec{r}(t) = \langle x(t), y(t)\rangle$ has $\vec{r}(0) = 0$. Suppose we additionally know that

$$\frac{d}{dt} f(x(t), y(t))\Big|_{t=0} = 6$$

Compute $|\vec{v'}(0)|$.

0.5. Estimate the Gradient at each of the points. Each grid length is 1 unit.



0.6. A. n ant travels in $x-y$ coordinates along the path $(3t, t^2)$ from time 0 to 2. It walks along the hill $f(x,y) = 2 - x^2 - y$ during this time.

- How long is the path that the ant travels along, and what is its maximal speed?
- When (if ever) does the ant travel perpendicular to the gradient of the hill?