Discussion Problems, Gradients and Directional Derivatives

0.1. **Gradient Calculation.** Jack and Jill walk up the hill $f(x, y)$, to fetch a pail of water. The path Jack takes is

$$r(t) = (3t, 2t)$$

and the path that Jill takes is

$$s(t) = (-2t, 3t)$$

Jack reports that at time 0,

$$\frac{d}{dt}(f(3t, 2t))\bigg|_{t=0} = 1$$

and Jill reports that

$$\frac{d}{dt}(f(-2t, 3t))\bigg|_{t=0} = 2$$

- What is the gradient $\nabla f(0, 0)$.
- Suppose additionally that $f(0, 0) = 2$. What is the tangent plane to the graph of f at $(0, 0, 2)$?

0.2. **Directional Derivatives of the Monkey Saddle.** The pointy Monkey saddle is given by the function

$$f(x, y) = \frac{y^3 - 3x^2y}{x^2 + y^2}$$

In polar coordinates, this is given by $r\cos(3\theta)$.

- Let $\vec{v} = \langle \cos\theta, \sin\theta \rangle$. Compute the directional derivative

$$D_{\vec{v}} f|_{(0,0)}.$$

  Hint: What should the directional derivatives at the origin of a function in polar form be?
- Using the above, compute the *Gradient* of $f(x, y)$ at the origin.
- Show that it is *not* the case that $D_{\vec{v}} = \nabla f \cdot \vec{v}$.
- What went wrong?