

QUIZ, FEB 22

NAME:

0.1. **Continuity.** Prove or disprove the continuity of the following function:

$$f(x, y) = \frac{x^2 + 2xy + y^2}{x^2 + y^2}$$

0.2. **Partial Derivatives, I.** Using partial derivatives at $(0, 0)$, estimate the value of

$$f(x, y) = \cos(x) \sin(y)$$

at the point $f(1/2, 1)$.

0.3. **Chain Rule.** Use the chain rule, and the function $f(x, y) = x/y$ to show the quotient rule:

$$\frac{d}{dt} \left(\frac{x(t)}{y(t)} \right) = \frac{y(t)x'(t) - x(t)y'(t)}{(y(t))^2}$$

Your proof may *not* use the chain rule. You may use the power rule in your proof.

Bonus Problem. *Worth no points!* Can you find $f(x, y)$, a function of two variables, so that

- $f(0, 0) = 0$
- $\lim_{t \rightarrow 0} f(at, bt) = 0$ for all a, b . (This means the limit value of $f(x, y)$ along every line approaching the origin is 0.)
- f is *not* continuous at $(0, 0)$!