

1. DERIVATIVES OF MULTI-VARIABLE FUNCTIONS, I

1.1. **Multivariable Chain Rule I.** Suppose that  $f(x, y)$  is a differentiable function of two variables, and we know its differential at a point to be:

$$(df)(1, 1) = 2dx + 3dy.$$

Compute  $\frac{d}{dt}f(t + 1, t^2 + 1)|_{t=0}$ .

1.2. **Multivariable Chain Rule II.** The product rule in single variable calculus states that  $(fg)' = f'g + g'f$ . Derive this using the chain rule from multivariable calculus. (This is an important exercise! Make sure you are able to identify at every step exactly what each derivative you are writing down means.)

1.3. **Differential I.** Is it possible for the differential of a function to be

$$df = (2x + 3)dx + (2x + 3)dy$$

If so, find a function which has this differential. If not, why?