Quiz, Feb 15

Name:

For most accurate results, try doing this without a textbook and spend no more than 15-20 minutes...

0.1. **Vector Valued Functions.** At $t = 0$, the vector valued functions

$$\vec{r}(t) = \langle e^t, t, t^2\rangle$$
$$\vec{s}(t) = \langle -t+1, t, t^3\rangle$$

intersect each other. What is the angle of their intersection?
Solution: The velocity of the first function is $\langle e^t, 1, 2t\rangle$. The velocity to the second one is $\langle -1, 1, 3t\rangle$. So, their angle at zero is related to the dot product of

$$\langle 1, 1, 0\rangle \cdot \langle -1, 1, 0\rangle = 0$$

so they are at right angles.

0.2. **Vector Valued Functions, II.** The helix is drawn out by the function $\vec{r}(t) = \langle \sin t, \cos t, t\rangle$. What is the arc length of the curve over the range $0 \leq t \leq 2\pi$.
Solution: We should integrate the length of the velocity vector. Here,

$$\vec{r'}(t) = \langle -\cos t, \sin t, 1\rangle$$

so that $|\vec{r'}(t)| = \sqrt{2}$. Then

$$\int_0^2 \pi |\vec{r'}| dt = \int_0^{2\pi} \sqrt{2} dt = \sqrt{2} 2\pi.$$

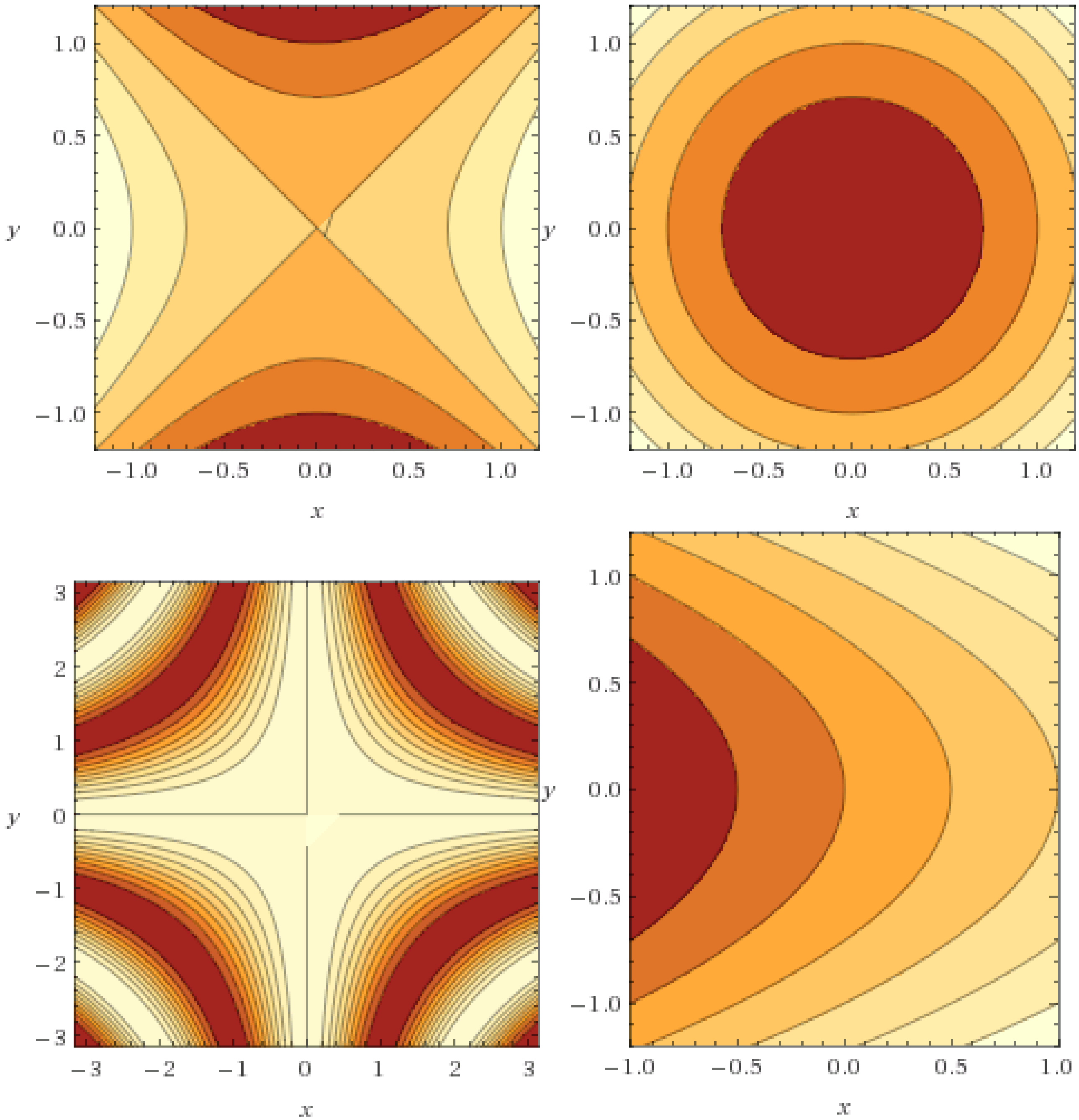
Bonus Problem. *Worth no points!* A small rocket is tied to a stick which is one meter long. The other end of the stick is tied to the origin. The rocket travels in a path $\vec{r}(t)$ – which is only confined by the stick. Geometrically justify and mathematically prove the following relation:

$$\vec{r'}(t) \cdot \vec{r}(t) = 0.$$

Solution: Since $\vec{r} \cdot \vec{r} = 1$ (that's the condition at the start) we apply the product rule in the derivative at get $\vec{r'} \cdot \vec{r} = 0$. This means that velocity is at right angles to the position, which makes sense, because the velocity should be tangent to the sphere of radius one (which is perpendicular to the radius, which gives the position of the rocket.)

0.3. **Functions and Contour Plots.** Quick! Match up the following graphs with their contour plots.

- $z = \cos(xy)$
- $z = y^2 + x$
- $z = x^2 - y^2$
- $z = x^2 + y^2$



Solution:

- $z = \cos(xy)$ is SW. Think what level sets $xy = \cos^{-1}(c)$ look like – the same as $y = (\cos^{-1}(c))/x$
- $z = y^2 + x$ is the SE. It looks like a bunch of shifted parabolas
- $z = x^2 - y^2$ NW. Think that $x^2 - y^2 = 0$ is the same as $x^2 = y^2$ is the same as $x = \pm y$.
- $z = x^2 + y^2$ is NE. The level sets look like circles of radius $\sqrt{c}$.