Groupwork I: Exploring the Monkey Saddle

My favorite function of two variables is called the monkey saddle. It is given by the following function:

$$f(x, y) = x(x^2 - 3y^2)$$

Solution: You should get the plane split into 6 equal size regions. The regions alternate positive and negative, starting with the region that contains the positive x axis. Find the $f(x, y) = 0$. From this information, start to make a contour plot: Which areas of the contour are bigger than 0? Which areas of the contour are smaller? Once you have finished this, consult with your teammates on why this surface is called the Monkey Saddle.

Groupwork II: Exploring the Monkey Saddle

My favorite function of two variables is called the monkey saddle. It is given by the following function:

$$f(x, y) = x(x^2 - 3y^2)$$

Find the intersection of the surface given by this graph with the xz plane.
Solution: The graph is of the cubic $z = x^3$. Once you have finished this, consult with your teammates on why this surface is called the Monkey Saddle.

Groupwork III: Exploring the Monkey Saddle

My favorite function of two variables is called the monkey saddle. It is given by the following function:

$$f(x,y) = x(x^2 - 3y^2)$$

Show that this has 3-way symmetry about the origin. Show that if $x = r\cos\theta$ and $y = r\sin\theta$, then $f(x,y) = r^3\cos(3\theta)$. Conclude that the Monkey Saddle graph has 3 fold symmetry around the origin by rotation.

Solution: Using the triple angle formula, we get that $r^3\cos(3\theta) = r^3\cos^3\theta + 3r^3\sin^2\theta\cos\theta$. This shows the equality given in the hint. Notice that $r^3\cos(3\theta) = r^3\cos(\theta + 2\pi n/3)$, which tells us that graph is symmetric under rotation by 120 degrees.

Once you have finished this, consult with your teammates on why this surface is called the Monkey Saddle.