## Discussion Problems, Vector Valued Functions I

(1) Find the acceleration at time 3 of the vector valued function

$$\vec{r}(t) = \langle 1 + \sin t, \sin t, 1\rangle$$

**Solution:**Taking the second derivative gives

$$\vec{r}''(t) = \langle 1 - \sin t, -\sin t, 0\rangle$$

so the accelration vector at time 3 is

$$\vec{r}''(3) = \langle 1 - \sin 3t, -\sin 3t, 0\rangle$$

(2) Find the velocity vectors and positions where these two vector valued functions intersect:

$$\vec{r}(t) = \langle 1 + t, t + 2, 2t^2 + 1\rangle$$

$$\vec{s}(t) = \langle 3 + t, t^2 + 2, t + 4\rangle$$

**Solution:**The point of intersection is where there are different values $t_0$ and $t_1$ such that

$$\vec{r}(t_0) = \vec{s}(t_1).$$

Checking the first coordinate, we have that $1 + t_0 = 3 + t_1$, so we know that $t_0 = 2 + t_1$. Taking the second coordinate, we have $t_0 + 2 = t_1^2 + 2$, so $2 + t_1 + 2 = t_1^2 + 2$, which tells us that $t_1^2 - t_1 - 2 = 0$, so that $t_1 = -1$ or $t_1 = 2$. On the last equation, we see that only $t_1 = -1$ is a valid solution. So we know that

$$\vec{r}(1) = \vec{s}(-1).$$

Checking the derivatives at this point, we have

$$\vec{r}'(t) = \langle 1, 24t\rangle$$

$$\vec{s}'(t) = \langle 1, 2t, 1\rangle$$

so that at these two times, there velocity vectors are $\vec{r}'(1) = \langle 1, 2, 4\rangle$ and $\vec{s}' = \langle 1, -2, 1\rangle$ respectively.

(3) Find the plane which contains both the velocity vector to $\vec{r}(t)$ and the velocity vector to $\vec{s}(t)$ at their point of intersection.

**Solution:**We know two vectors in the plane, so the normal to the plane will be given by their cross product. Their cross product is

$$\vec{N}_p = \vec{r}'(1) \times \vec{s}'(-1) = (10, 3, -3)$$

Plugginging a point into the equation of a plane that we know, $\vec{r}(1) = \langle 2, 3, 5\rangle$

$$10x + 3y - 3z = d$$

we get that $d = 2 \cdot 10 + 3 \cdot 3 - 3 \cdot 5 = 14$.

(4)

(5) Show that the function

$$\vec{r}(t) = \langle 1 + t^2, 1 + t^2, t\rangle$$

does not intersect the plane

$$-2x + 3y + z = 1.$$

Then find the closest point of the vector valued function to the plane by two methods:

- Finding where the velocity of $\vec{r}(t)$ is parallel to the plane
- Taking the distance function between a point and the plane, and minimizing it.

Are these two always going to be the same?

**Solution:**To show that they do not intersect, plug the formula of the curve into the formula for the plane component wise. Since $-2(1 + t^2) + 3(1 + t^2) + 1(1 + t)$ is always greater than 1, so the plane cannot intersect the curve.

The velocity vector of the curve is given by $\vec{r}'(t) = \langle 2t, 2t, 0\rangle$. We want to see when this is parallel to the plane. This occurs when this velocity vector is perpendicular to the normal vector of the plane. So, we solve for $t$ such that

$$\vec{r}'(t) \cdot \vec{N}_p = 0$$

Substituting, we get

$$\langle 2t, 2t, 1\rangle \cdot \langle -2, 3, 1\rangle = 0$$

Solving for this, we see that $t = -1/2$.
For the second part, we can you the "plane point distance formula"

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

and substitute our curve in for the values $\langle x_0, y_0, z_0\rangle$. From this we get

$$D(t) = \frac{|-2(1+t^2) + 3(1+t^2) + 1(1+t) - 1|}{\sqrt{(-2)^2 + 3^2 + 1^2}} = \frac{t^2 + t + 1}{\sqrt{14}}$$

Removing the absolute value signs, and taking the derivative, we need to find the value of $t$ making $2t + 1 = 0$, which is when $t = \frac{-1}{2}$.