(1) Find the acceleration at time 3 of the vector valued function
\[ \mathbf{r}(t) = \langle 1 + \sin t, \sin t, 1 \rangle \]

**Solution:** Taking the second derivative gives
\[ \mathbf{r}''(t) = \langle 1 - \sin t, -\sin t, 0 \rangle \]
so the acceleration vector at time 3 is
\[ \mathbf{r}''(3) = \langle 1 - \sin 3t, -\sin 3t, 0 \rangle \]

(2) Find the velocity vectors and positions where these two vector valued functions intersect:
\[ \mathbf{r}(t) = \langle 1 + t, t^2 + 2, t^2 + 1 \rangle \]
\[ \mathbf{s}(t) = \langle 3 + t, t^2 + 2, t + 4 \rangle \]

**Solution:** The point of intersection is where there are different values \( t_0 \) and \( t_1 \) such that
\[ \mathbf{r}(t_0) = \mathbf{s}(t_1) \]
Checking the first coordinate, we have that \( 1 + t_0 = 3 + t_1 \), so we know that \( t_0 = 2 + t_1 \). Taking the second coordinate, we have \( t_0 + 2 = t_1^2 + 2 \), so \( 2 + t_1 + 2 = t_1^2 + 2 \), which tells us that \( t_1^2 - t_1 - 2 = 0 \), so that \( t_1 = -1 \) or \( t_1 = 2 \). On the last equation, we see that only \( t_1 = -1 \) is a valid solution. So we know that
\[ \mathbf{r}(1) = \mathbf{s}(-1) \]
Checking the derivatives at this point, we have
\[ \mathbf{r}'(t) = \langle 1, 24t \rangle \]
\[ \mathbf{s}'(t) = \langle 1, 2t, 1 \rangle \]
so that at these two times, their velocity vectors are \( \mathbf{r}'(1) = \langle 1, 24 \rangle \) and \( \mathbf{s}' = \langle 1, -2, 1 \rangle \) respectively.

(3) Find the plane which contains both the velocity vector to \( \mathbf{r}(t) \) and the velocity vector to \( \mathbf{s}(t) \) at their point of intersection.

**Solution:** We know two vectors in the plane, so the normal to the plane will be given by their cross product. Their cross product is
\[ \mathbf{N}_p = \mathbf{r}'(1) \times \mathbf{s}'(-1) = (10, 3, -3) \]
Plugging a point into the equation of a plane that we know, \( \mathbf{r}(1) = (2, 3, 5) \)
\[ 10x + 3y - 3z = d \]
we get that \( d = 2 \cdot 10 + 3 \cdot 3 - 3 \cdot 5 = 14 \).

(4) Show that the function
\[ \mathbf{r}(t) = \langle 1 + t^2, 1 + t^2, t \rangle \]
does not intersect the plane
\[ -2x + 3y + z = 1. \]
Then find the closest point of the vector valued function to the plane by two methods:
- Finding where the velocity of \( \mathbf{r}(t) \) is parallel to the plane
- Taking the distance function between a point and the plane, and minimizing it.
Are these two always going to be the same?

**Solution:** To show that they do not intersect, plug the formula of the curve into the formula for the plane component wise. Since \( -2(1 + t^2) + 3(1 + t^2) + 1(1 + t) \) is always greater than 1, so the plane cannot intersect the curve.
The velocity vector of the curve is given by \( \mathbf{r}'(t) = (2t, 2t, 0) \). We want to see when this is parallel to the plane. This occurs when this velocity vector is perpendicular to the normal vector of the plane. So, we solve for \( t \) such that
\[ \mathbf{r}'(t) \cdot \mathbf{N}_p = 0 \]
Substituting, we get

\[ \langle 2t, 2t, 1 \rangle \cdot \langle -2, 3, 1 \rangle = 0 \]

Solving for this, we see that \( t = -1/2 \).

For the second part, we can use the “plane point distance formula”

\[ D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \]

and substitute our curve in for the values \( \langle x_0, y_0, z_0 \rangle \). From this we get

\[ D(t) = \frac{|-2(1 + t^2) + 3(1 + t^2) + 1(1 + t) - 1|}{\sqrt{(-2)^2 + 3^2 + 1^2}} = \frac{t^2 + t + 1}{\sqrt{14}} \]

Removing the absolute value signs, and taking the derivative, we need to find the value of \( t \) making \( 2t + 1 = 0 \), which is when \( t = -\frac{1}{2} \).