Name:

0.1. Cross Product. Compute the following Cross Products:

$$\langle 1, 2, 1 \rangle \times \langle 1, 4, 5 \rangle$$
 $\langle 7, -1, 3 \rangle \times \langle -14, 2, -6 \rangle$

Solution: The first cross product is given by

$$\langle 1, 2, 1 \rangle \times \langle 1, 4, 5 \rangle = \begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \hat{k}$$
$$= (10 - 4)\hat{i} - (5 - 1)\hat{j} + (4 - 2)\hat{k}$$
$$= \langle 6, -4, 2 \rangle$$

For the second cross product, notice that the second vector is -2 times the first one. Therefore, the cross product is zero (as the vectors are parallel.)

0.2. **Lines and Planes.** Find the equation of a plane which is parallel to the plane 2x - 2y + z = 2 and is distance 6 away.

Solution:Since our new plane is parallel to the old one, we know that the normal vector to each plane is the same. Let's call this vector $\vec{N} = \langle 1, 2, 1 \rangle$. So we know the new plane has equation

$$2x - 2y + z = c$$

for some different value c. We just need to find a point which is distance 6 from the old plane and plug it in to find the value of c. Take any point you want on the plane– like (0, 0, 2), then add a normal vector of length two. The corresponding unit normal vector is $\hat{N} = \langle 2/3, -2/3, 1/3 \rangle$. So a point distance 6 from the old plane is

$$(0,0,2) + 6\hat{N} = (4,-4,4)$$

Plugging this in, we get

$$2x - 2y + z = 20$$

0.3. **Projections.** Show that $Comp_{\vec{u}}(\vec{v} \times \vec{u}) = 0$.

Solution: The geometric explanation is that \vec{u} is perpendicular to $\vec{v} \times \vec{u}$, so the component of one onto the other is going to be zero.

Computationally,

$$\begin{aligned} \operatorname{Comp}_{\vec{u}}(\vec{v} \times \vec{u}) &= \frac{\vec{v} \cdot (\vec{v} \times \vec{u})}{\vec{u}} \\ &= \frac{1}{|\vec{v}|} \langle v_x, v_y, v_z \rangle \cdot \langle v_y u_z - v_z u_y, -(v_x u_z - v_z u_x), v_x u_y - v_y u_x \rangle \\ &= v_x v_y u_z - v_x v_z u_y - v_y v_x u_z + v_y v_z u_x + v_z v_x u_y - v_z v_y u_x \\ &= 0 \end{aligned}$$

Bonus Problem. Worth no points! Let $F_1, \ldots F_k$ describe the faces of a polyhedron. Let $\vec{N}_1, \ldots \vec{N}_k$ be the outward pointing normal vectors to these faces, with $|\vec{N}_i| = \text{Area}(F_i)$. Prove that for any vector \vec{b} , we have

$$\vec{b} \cdot \vec{N}_1 + \vec{b} \cdot \vec{N}_2 + \dots + \vec{b} \cdot \vec{N}_k = 0$$

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