Worksheet, Feb 1

0.1. **Component of vectors.** Find a unit vector that points in the same direction as

$$\vec{v}\langle 1,3,1\rangle$$

Find the component of the vector $\vec{u}=\langle 2,1,1\rangle$ onto this vector.
Solution: For the first part, we know that the unit vector that points in the $\vec{v}$ direction is $\vec{v}/|\vec{v}|$. In this case we have the vector

$$\frac{1}{\sqrt{1^2+3^2+1^2}}\langle 1,3,1,\rangle = \langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\rangle$$

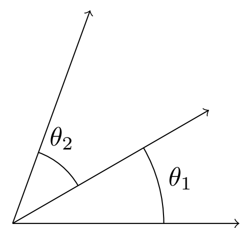
The component of $\vec{u}$ onto $\vec{v}$ is given by

$$\mathrm{Comp}_{\vec{v}}\vec{u} = \frac{\vec{v}\cdot\vec{u}}{|\vec{v}|} = \frac{2\cdot 1+1\cdot 3+1\cdot 1}{\sqrt{11}} = \frac{6}{\sqrt{11}}$$

0.2. **Angles.** Prove the identity

$$\cos(\theta_1+\theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

by using the dot product formula for angles and the following setup of vectors.



Solution: We use both the formula for sin and cosine from determinant and dot product respectively. Let's call the vectors (from bottom to top) $\vec{a},\vec{b},\vec{c}$, and assume that they are unit length. Then we have

$$\begin{aligned}\sin\theta_1 =& \det(\vec{a},\vec{b})\\ \sin\theta_2 =& \det(\vec{b},\vec{c})\\ \cos\theta_1 =& \vec{a}\cdot\vec{b}\\ \cos\theta_2 =& \vec{b}\cdot\vec{c}\\ \cos(\theta_1+\theta_2) =& \vec{a}\cdot\vec{c}\end{aligned}$$

Then we compute

$$\begin{aligned}(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c}) - \det(\vec{a},\vec{b})\det(\vec{b},\vec{c}) =& (a_xb_x+a_yb_y)(b_xc_x+b_yc_y) - (a_xb_y-a_yb_x)(b_xc_y-b_yc_x)\\ =& (a_xb_xb_yc_y + a_yb_yb_xc_x + a_xb_xb_xc_x + a_yb_yb_yc_y)\\ &- (a_xb_xb_yc_y - a_xb_yb_yc_x - a_yb_xb_xc_y + a_yb_xb_yc_x)\end{aligned}$$

Some terms cancel

$$\begin{aligned}=& (a_xb_xb_xc_x + a_yb_yb_yc_y + a_xb_yb_yc_x + a_yb_xb_xc_y)\\ =& a_xc_x(b_xb_x+b_yb_y) + a_yc_y(b_xb_x+b_yb_y)\\ =& (a_xc_x+a_yc_y)|\vec{b}|\\ =& \vec{a}\cdot\vec{c}\\ =& \cos(\theta_1+\theta_2)\end{aligned}$$