QUIZ, FEB 1

NAME:

0.1. **Lengths of Curves.** Find the length of the curve given by the following parametric equations where $1 \le t \le 3$

$$\begin{aligned} x(t) =& \frac{t^2}{2} \\ y(t) =& (2t+9)^{\frac{3}{2}} \end{aligned}$$

Solution: We use the length formula

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

We have that

$$\frac{dx}{dt} = t \qquad \frac{dy}{dt} = 3\sqrt{2t+9}$$

Plugging in we get

$$\begin{aligned} L =& \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ =& \int_1^3 \sqrt{(t)^2 + \left(3\sqrt{2t+9}\right)^2} dt \\ =& \int_1^3 \sqrt{(t)^2 + 18t + 81} dt \\ =& \int_1^3 \sqrt{(t+9)^2} dt \\ =& \int_1^3 \sqrt{t+9} dt \\ =& \frac{t^2}{2} + 9t \Big|_1^3 \\ =& \frac{9}{2} + 27 - \frac{1}{2} - 9 = 22 \end{aligned}$$

0.2. **Polar Graphs.** Graph the polar equation $r(\theta) = \frac{1}{\sqrt{2}}(\cos(\theta) + \sin(\theta))$ where θ goes from 0 to π.
Solution: Three ways to do this:

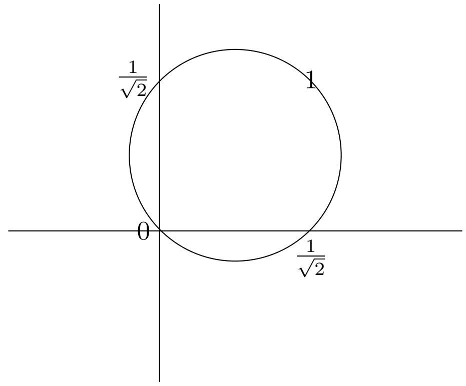
(1) Calculate some values for $r(\theta)$ and graph them. If you choose

$$r(0) = \frac{1}{\sqrt{2}} \qquad r(\pi/4) = 1$$

$$r(\pi/2) = \frac{1}{\sqrt{2}} \qquad r(3\pi/2) = 0$$

$$r(\pi) = \frac{-1}{\sqrt{2}}$$

And plot those points, you get the following picture:



Notice here that $r(\pi)$ plots the same point as $r(0)$. This is because the radius value is *negative*

(2) You can convert Cartesian coordinates by

$$\begin{aligned} r(\theta) =& \frac{1}{\sqrt{2}}\left(\cos(\theta)+\sin(\theta)\right) \\ r(\theta)^2 =& \frac{1}{\sqrt{2}}\left(r(\theta)\cos\theta + r(\theta)\sin(\theta)\right) \\ x^2+y^2 =& \frac{1}{\sqrt{2}}\left(x+y\right) \end{aligned}$$

This is the equation for a circle centered at $(1/\sqrt{2}, 1/\sqrt{2}$ with a radius of $1/2$.

(3) You can use the trig identity $\cos(\theta+\theta_1) = \cos(\theta)\cos(\theta_1) - \sin(\theta)\sin(\theta_1)$. By letting $\theta_1 = -\pi/4$, we have

$$r(\theta) = \cos(\theta - \pi/4).$$

As $\tilde{r}(\theta) = \cos(\theta)$ plots a circle of radius $1/2$ centered at $(1/2, 0)$– from lecture – we know that this graphs the same circle, just everything is rotated.

0.3. **Vector Geometry.** Use vectors to show that diagonals of a square are perpendicular to each other.
Solution: Almost everybody got the right solution to this, so I am not going to type up the solution. If you have a question about this problem, come ask me during office hours!

Bonus Problem. Suppose that $\vec{a}_i$ are a bunch of vectors forming the boundary of a polygon. Show that for any vector $\vec{b}$, we have

$$\vec{b} \cdot \vec{a}_1 + \vec{b} \cdot \vec{a}_2 + \cdots \vec{b} \cdot \vec{a}_n = 0$$

Solution:

$$\vec{b} \cdot \vec{a}_1 + \vec{b} \cdot \vec{a}_2 + \cdots \vec{b} \cdot \vec{a}_n = \vec{b} \cdot (\vec{a}_1 + \vec{a}_2 + \cdots \vec{a}_n)$$

Because the $\vec{a}_i$ close up to a cycle when placed head - to -tail

$$= \vec{b} \cdot 0$$
$$= 0$$

On physical interpretation of this is the following. Imagine thata you had a river, which flowed in the direction of $\vec{b}$. Then draw a polygon with the vectors $\vec{a}_i$ somewhere in the river. The amount of flow of water through the i face of the polygon is $\vec{b} \cdot \vec{i}$. Therefore, the sum in question computes the amount

$$\text{water entering the polygon} - \text{water leaving the polygon}$$

This should be zero, because there isn't any water accumlating inside of the polygon.