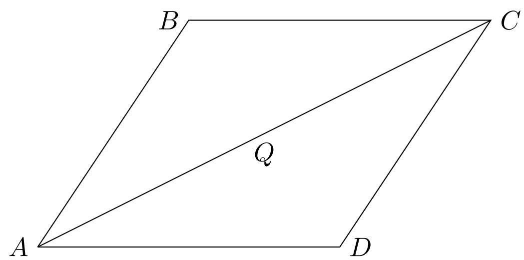
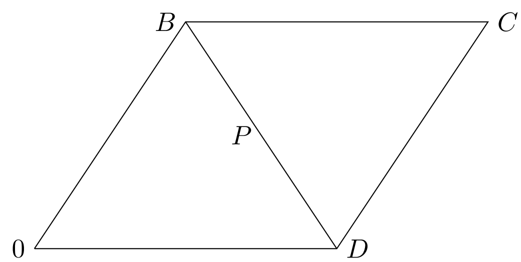
0.1. **Some Simple Vector Manipulations.** Prove that the diagonals of a parallelogram intersect in their midpoint.



Let's first calculate the position of the point Q, which is the midpoint of the vector $\vec{AC}$. Notice that

$$\vec{AQ} = \frac{1}{2}\vec{AC} = \frac{1}{2}\vec{AB} + \vec{BC}$$

What about if we were to find the midpoint of the other diagonal?



This lies as

$$\vec{DQ} = \frac{1}{2}\vec{DB} = \frac{1}{2}(\vec{DC} - \vec{BC})$$

We have that

$$\begin{aligned}\vec{AQ} =& \vec{AD} + \vec{DQ}\\ =& \vec{AD} + \frac{1}{2}(\vec{DC} - \vec{BD})\end{aligned}$$

By the definition of paralleogram, $\vec{DC} = \vec{AB}$

$$=\vec{AD} + \frac{1}{2}(\vec{AB} - \vec{BC})$$

Also, $\vec{AD} = \vec{BC}$

$$\begin{aligned}=& \vec{BC} + \frac{1}{2}\vec{AB} - \frac{1}{2}\vec{BC}\\ =& \frac{1}{2}\vec{AB} + \vec{BC}\\ =& \vec{AQ}\end{aligned}$$

So they lie at the same point.

0.2. **A Triangle Computation.** Let $A = (1, 2, 1)$, $B = (0, -1, 2)$ and $C = (3, 1, 3)$.

- Write the vectors $\vec{AB}$, $\vec{BC}$ and $\vec{CA}$.

$$\vec{AB} = (0, -1, 2) - (1, 2, 1) = \langle -1, -3, 1 \rangle$$
$$\vec{BC} = (3, 1, 3) - (0, -1, 2) = \langle 3, 2, 1 \rangle$$
$$\vec{CA} = (1, 2, 1) - (3, 1, 3) = \langle -2, 1, -2 \rangle$$

- What are these vectors in components?

$$\vec{AB} = -\hat{\imath} - 3\hat{\jmath} + \hat{k}$$
$$\vec{BC} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$$
$$\vec{CA} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$$

- What are the lengths of these vectors?

$$|\vec{AB}| = \sqrt{(-1)^2 + (-3)^2 + 1^2} = \sqrt{11}$$
$$|\vec{BC}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$
$$|\vec{CA}| = \sqrt{(-2)^2 + 1^2 + (-2)^2} = \sqrt{9} = 3$$

- What is the angle containing the point B in the triangle ABC?
  Here it is important to notice that we are looking for the angle between $\vec{B}A$ and $\vec{B}C$.

$$|\vec{BC}||\vec{BA}| \cos\theta = \vec{BC} \cdot \vec{BA}$$
$$\sqrt{14} \cdot \sqrt{11} \cos\theta = (1 \cdot 3) + (3 \cdot 2) + (1 \cdot 1)$$
$$\sqrt{154} \cos\theta = 10$$
$$\theta = \cos^{-1}\left(\frac{10}{\sqrt{154}}\right)$$
$$\sim 36.3 \text{degrees}$$