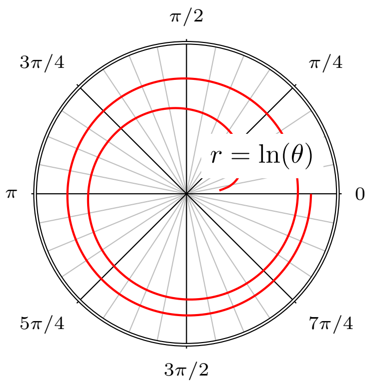
Discussion Examples, Jan 28

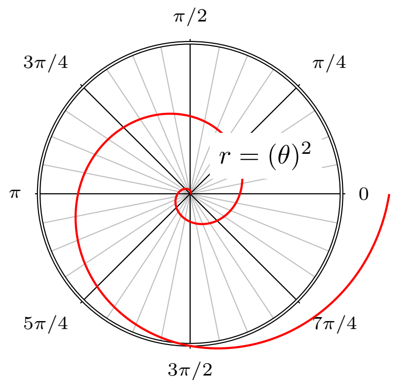
1. Graphing Polar Equations

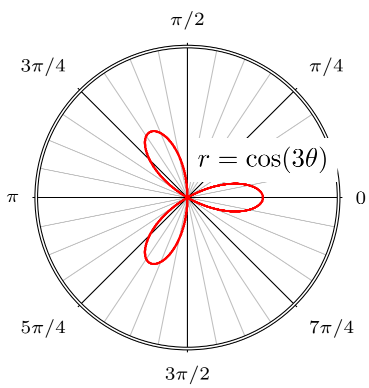
Problem 1.3.54) Match the polar equations with the graphs. The equations are:

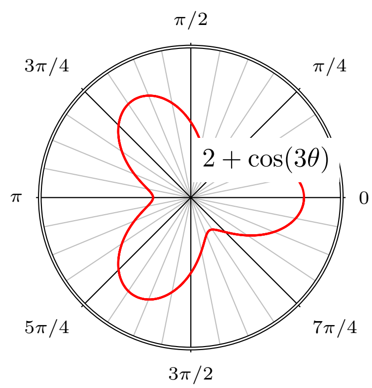
- $r = \ln\theta$
- $r = \theta^2$
- $r = \cos 3\theta$
- $r = 2 + \cos 3\theta$
- $r = \cos(\theta/2)$
- $r = 2 + \cos(3\theta/2)$.

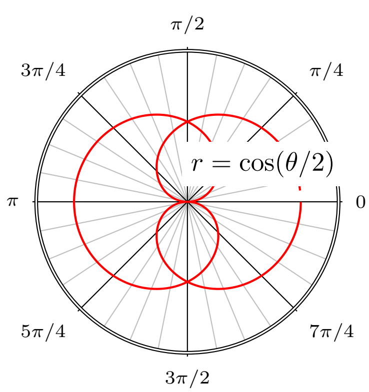
Here are the graphs:

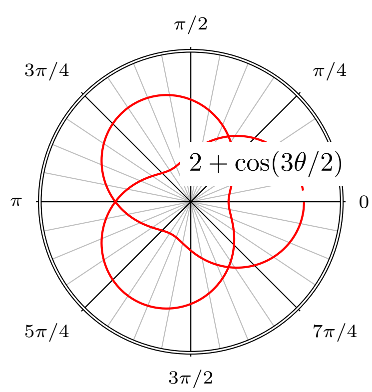












2. Intersections

Problem 10.4.41 Find the intersections of $r_1 = \sin\theta, \;\; r_2 = \sin 2\theta$.
To find the intersection of these two curves, we want to find values of θ that make the values of r equal to eachother. One thing to be careful about it that there can be an intersection between to curves r_1 and r_2 whenever $r_1(\theta) = r_2(\theta + 2\pi n)$, or $r_1(\theta) = -r_2(\theta + \pi + 2\pi n)$.
In this case, we can finding the intersection between a clover and a circle. From drawing out the curves, we expect to find 3 intersections. Setting $r_1 = r_2$ we have

$$\sin\theta = \sin(2\theta)$$

Using the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$$\begin{aligned}\sin\theta =& 2\sin(\theta)\cos(\theta)\\ (1 - 2\cos(\theta))\sin(\theta) =& 0\end{aligned}$$

This occurs either when $\sin(\theta) = 0$, or $1 - 2\cos(\theta) = 0$. Working this out, we get that $\theta = 0, \theta = \pm\pi/3$. Checking those values give us the intersection points.

2.1. Area Enclosed.

10.4. Example 1 Find the area enclosed by one leaf of the clover $r = \cos 2\theta$.
Based on finding the self intersections, we know that the bounds of the integral are from $-\pi/4$ to $\pi/4$. Then the integral formula gives us

$$\begin{aligned}A =& \int_{-\pi/4}^{\pi/4} \frac{1}{2} r^2 d\theta\\ =& \frac{1}{2}\int_{-\pi/4}^{\pi/4} \cos^2 2\theta d\theta\\ =& \int_0^{\pi/4} \cos^2 2\theta d\theta\\ =& \int_0^{\pi/4} (\frac{1}{2}(1 + \cos 4\theta) d\theta\\ =& \frac{1}{2}\left(\theta + \frac{1}{4}\sin 4\theta\right)\end{aligned}$$

Evalutated on endpoints gives us

$$= \pi/8$$