1. Tangents, Arclengths an Polar Coordinates

The spiral is graphed out by the parametric equation

$$x(t) = t\cos t \quad y(t) = t\sin t$$

where $t \geq 0$

(1) What is equation for the line which touches the point on the curve $(0, \pi/2)$?
Solution: To do this, we need to determine the slope of the curve at this point. The slope is given by

$$\frac{dy}{dx} = \frac{y'(t)}{x(t)} = \frac{\sin t + t\cos t}{\cos t - t\sin t}$$

The point lies on the curve at $t = \pi/2$. The value of the slope when $t = \pi/2$ is $\frac{1}{-\pi/2} = -2/\pi$. The equation of the line is therefore

$$y = -2/\pi x + \pi/2.$$

(2) Write down an integral which computes the length of a spiral from $t = 0$ to $t = 2\pi$.
Solution: The function is

$$\begin{aligned} L &= \int_0^{t_{fin}} \sqrt{\left(\frac{dy}{dt}\right))^2 + \left(\frac{dx}{dt}\right)^2} dt \\ &= \int_0^{t_{fin}} \sqrt{(\sin t + t\cos t))^2 + (\cos t - t\sin t)^2} dt \\ &= \int_0^{t_{fin}} \sqrt{1+t^2} dt \end{aligned}$$

This integral can be solved, but it is a little tricky.

(3) The Limacon is graphed by the polar equation

$$r = 1 + c\sin\theta$$

- What shape is this graph when $c = 0$?
  Solution: A circle

- Sketch a graph of this when $c = 1$.
  Solution:
- What is the graph of $r = \cos(\theta)$ look like?
  Solution: It is a circle, but centered at the point $(.5, 0)$.
- Describe how the Limacon changes as c goes to infinity.
  Solution: It looks more and more like a circle centered at the point $(c/2, 0)$.