0.1. **Example One: Slope, Area and Arclength and Surface Area under a circle.** An easy problem where we already know the solution.

- The parametric equation for a circle is given by
$$x(t) = r\cos t \qquad\qquad y(t) = r\sin t$$
and we let t vary from 0 to 2π.
- The slope of a tangent line to a parametric curve at time t is given by
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$
In this case, we compute
$$\begin{aligned}\frac{dy}{dt} &= r\cos t\\ \frac{dx}{dt} &= -r\sin t\\ \frac{dy}{dx} &= \frac{r\cos(t)}{-r\sin(t)} = -\tan(t)\end{aligned}$$
You can verify that this is the slope of the line geometrically using trigonometry.
- To calculate area under a cartesian curve we integrate:
$$A = \int_a^b y dx$$
By using the substitution $y = y(t)$ and $dx = \frac{dx}{dt}dt$ we can make this an area formula for parametric curves:
$$A = \int_{t_0}^{t_1} y(t)\frac{dx}{dt}dt$$
where t_0 and t_1 mark the start and ending of the curve. Notice that this only makes sense if $x'(t) \geq 0$ for all t or $x'(t) \leq 0$ for all t– the curve can't back around on itself.
Let's use this in the example of a circle again. Notice here that $x'(t) \leq 0$ when t runs from 0 to π. We'll compute the area under the semicircle, and double it.
$$\begin{aligned}A &= \int_0^\pi (r\sin t)(-r\sin t)\, dt\\ &= \int_0^\pi -r^2\sin^2 t dt\end{aligned}$$
Writing $\sin^2 t = \frac{1}{2}t - \frac{1}{2}\cos(2t)$
$$\begin{aligned}&= -r^2\int_0^{2\pi}\left(\frac{1}{2}t - \frac{1}{2}\cos(2t)\right)dt\\ &= -\frac{r^2}{2} + \frac{r^2}{4}\sin 2t \quad \text{evaluated from t=0 to } \pi\\ &= -\frac{r^2\pi}{2}\end{aligned}$$
Which is the area that we know for the upper semicircle.
- Finally, let's do an example of finding arclength. The arclength formula for a parametric curve is
$$L = \int_{t_0}^{t_1}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$$

In the case for the circle, this becomes

$$\begin{aligned}L =& \int_0^{2\pi} \sqrt{(-r\sin(t))^2 + (r\cos(t))^2} dt \\ =& \int_0^{2\pi} r dt \\ =& 2\pi t\end{aligned}$$