Discussion Problems, Parametric Curves II

(1) *10.1.37* Consider the parametric equations

$$x_1(t) = e^t \quad y_1(t) = e^{-2t}$$
$$x_2(s) = \cos(s) \quad y_2(s) = \sec^2 s$$

By converting each of these curves to a cartesian equation, show they represent the same curve.

Find an equation for t in terms of s that reparameterizes first equation into the second.

(2) Given two curves parametric equations, we can consider their *intersection points* and their *collision points*.

- An intersection point is where the two equations have the same x and y values, but possibly at different times.
- A collision point is where the two equations have the same x and y values at the same time.

Consider the curves given by

$$x_1 = 3\sin t \quad y_1 = 2\cos t$$
$$x_2 = t \quad y_2 = 0$$

Find any intersection or collision points that exist for these 2 equations.

Can you come up with a different parameterization of the second equation that turns the intersection points into collision points?

Is it always possible to do this? That is, given 2 parametric curves, can you always reparameterize them in such a way that every intersection point is a collision point?

(3) Consider the curve

$$x(t) = \cos(t) + \sin(t)$$
$$y(t) = \sin(3t)$$

When does this curve achieve a maximal x value?

Find the equation for a vertical line which is tangent to this curve.

By finding the maximum x values and y values, you can get a better idea of what a curve looks like. Graph the curve using this information.