Discussion Problems, Parametric Curves I

(1) Why is it that every curve given as the graph of $y = f(x)$ can be expressed as a parametric curve?

Solution: If $y = f(x)$ is the cartesian equation, take the parametric plot:

$$x(t) = t \quad y(t) = f(t).$$

Give an example of a parametric curve which cannot be expressed as the graph of $y = f(x)$ for any function $f(x)$.

Solution: Consider $x(t) = 0$ and $y(t) = t$. It is possible to represent a parametric curve as a cartesian curve whenever the function $x(t)$ is invertible.

(2) *10.1.18* Consider the parametric curve given by

$$x = \tan^2\theta, \quad y = \sec\theta$$

- Eliminate the parameter θ to find a Cartesian equation of the curve.

  Solution: Use the identity $\tan^2(\theta) + 1 = \sec^2(\theta)$. This gives you $x + 1 = y^2$, or $y = \pm\sqrt{x+1}$.

- Do you think it easier to graph the parametric curve, or the Cartesian one?

- Graph this parametric curve as θ varies between $-\pi/2$ and $\pi/2$.

(3) *10.1.45* Given two curves parametric curves, we can consider their *intersection points* and their *collision points.*

- An intersection point is where the two curves have the same x and y values, but possibly at different times.
- A collision point is where the two curves have the same x and y values at the same time.

Consider the curves given by

$$x_1 = 3\sin t \quad y_1 = 2\cos t$$
$$x_2 = -3 + \cos t \quad y_2 = 1 + \sin t$$

Find any intersection or collision points that exist for these 2 curves.
Can you come up with a different parameterization of the second curve that turns the intersection points into collision points?

Is it always possible to do this? That is, given 2 parametric curves, can you always reparameterize them in such a way that every intersection point is a collision point?