1. Consider the following system of linear equations:

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 5y + 10z &= 3 \\ -x + 2y - 2z &= 3 \end{aligned}$$

   (a) Using an inverse matrix, solve the above system of linear equations.
   (b) Find a basis for the nullspace for this system of linear equations
   (c) Find a basis for the column space of this system of linear equations.

   Solution: So the first part is to compute the inverse matrix. You may use any method you choose, (including the dreaded Cramer's rule) but it is porbably easies to use simple row operations.

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & -5 & 10 & 0 & 1 & 0 \\ -1 & 2 & -2 & 0 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & -2 & 0 & -3 \\ 0 & 1 & 0 & 6 & -1 & 4 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -2 & -5 \\ 0 & 1 & 0 & 6 & -1 & 4 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

   (a) Given the inverse matrix, we have that the solution should be

$$\begin{pmatrix} 10 & -2 & -5 \\ 6 & -1 & 4 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 20 - 6 - 15 \\ 12 - 3 + 4 \\ 2 + 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 13 \\ 5 \end{pmatrix}$$

   (b) Here is the matrix for rotation by 30 degrees

$$R_{30} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

   (c) Since the matrix is invertible, the nullspace is 0.
   (d) Since this is a square invertible matrix, the matrix is onto, so the column space is all of R^3, so a basis could be e_1, e_2, e_3.

   (a) Compute the matrix for rotation by 60 degrees.
   (b) What is the determinant for R_{120}?
   (c) Compute the inverse to R_{30}.

2. Find the area of the parallelpiped with corners at $(0,0,1), (1,1,0), (1,0,1)$ and $(2,-1,1)$

3. True or false!

   (a) Every matrix can be written as the product of elementary matricies.
   (b) Let U, W be subspaces of V. Then the **sum of U and W** is a subspace.
   (c) If A is a matrix, and A' is obtained from A by row opererations, then $\det A = \det A'$
   (d) If A has a pivot in every row, then it is one to one.
   (e) Suppose that $A : V \to W$ is a matrix, with the columns of A spanning W. Then $A\vec{x} = \vec{w}$ always has a solution.
   (f) Suppose that A and B are square matrices, and AB is not invertible. Then A is not invertible.
   (g) $(2,1,2)$ is in the nullspace of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$.