Practice Final 2

1. True or False!

    (a) If $A = A^T$, then A is diagonalizable.

    (b) If A and B have the same eigenvalues, then they are similar.

    (c) If y_1 and y_2 are solutions to a linear first order nonhomogenous differential equation, then y_1 is a multiple of y_2

    (d) If λ is a complex eigenvalue for a matrix A, then $\bar{\lambda}$, it's complex conjugate, is also a complex eigenvalue.

    (e) Suppose $W[y_1, y_2](t) = 0$ for all t. Then y_1 and y_2 are linearly dependent.

    (f) Suppose W_1, W_2 are subspaces of V. Then the union of W_1 and W_2 is a subspace of V.

    (g) Let v be a vector in a vector space V. Then the set of all vectors perpendicular to v is a subspace of V.

    (h) The set of all odd functions is a subspace of the space of continuous function from $\mathbb{R}$ to $\mathbb{R}$.

    (i) Every periodic function has a Fourier expansion.

    (j) Suppose A is a 3×3 matrix with eigenvalues $1, 2, 3$. Then A is diagonalizable.

Solution 1

(a) True!

(b) False! For example, the 0 matrix and the matrix with just a 1 in the upper right hand corner.

(c) False! If it were homogeneous, it would be true.

(d) True! If λ is a root of a polynomial, so is $\bar{\lambda}$.

(e) False! Take for instance x^2 and $x|x|$.

(f) False! It may not be closed under addition.

(g) True!

(h) True!

(i) False, take for instance secant or something nasty like that.

(j) True.

2. Find the general solution to the following differential equation:

$$y'' + 2ty' + y = te^{-t} + e^{-t}$$

Solution 2 First start by finding the solutions to the homogenous part. Solving the characteristic polynomial

$$\lambda^2 + 2\lambda + 1 = 0$$

yields a double root $\lambda = -1$. So we know that the solution to the homogenous part is of the form

$$y_h = c_1 e^{-t} + c_2 t e^{-t}$$

Now we have to make some guesses. Because the homogenous solution matches what we might normally guess, we have to bump everything up by a degree of 2. So our guess is going to be of the form

$$at^3 e^{-t} + bt^2 e^{-t}$$

When we plug this into our differential equation we get

$$\begin{aligned}a(6te^{-t} - 3t^2e^{-t} - 3t^2e^{-t} + te^3e^{-t}) + b(2e^{-t} - 2te^{-t} - 2te^{-t} + t^2e^{-t}) \\ +2(a(3t^2e^{-t} - t^3e^{-t}) + b(2te^{-t} - t^2e^{-t})) \\ at^3e^{-t} + bt^2e^{-t} = te^{-t} + e^{-t}\end{aligned}$$

Combining like terms gives us

$$(6a)te^{-t} + 2be^{-t} = te^{-t} + e^{-t}$$

or that $b = \frac{1}{2}$ and $a = \frac{1}{6}$.

3. Find the general solution to

$$y^{(4)} + 2y^{(3)} + 4y^{(2)} - 2y^{(1)} - 5y = 0$$

Solution 3 Guess that $e^{\lambda t}$ is a solution to this gives us the characteristic polynomial

$$\lambda^4 + 2\lambda^3 + 4\lambda^2 - 2\lambda - 5 = 0$$

This factors as

$$(\lambda + 1)(\lambda - 1)(\lambda + 2\lambda + 5)$$

The last part has roots $(-1 \pm 2i)$
This means that your fundamental solutions should be

$$\begin{aligned}y_1 &= e^{-t} \\ y_2 &= e^{t} \\ y_3 &= e^{-t}\cos(2t) \\ y_4 &= e^{-t}\sin(2t)\end{aligned}$$

and a general solution is of the form

$$y = c_1y_1 + c_2y_2 + c_3y_3 + c_4y_4$$

4. Find the matrix for orthogonal projection onto the subspace spanned by

$$u = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \qquad v = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$