QUIZ, NOV. 13

NAME:

Setting up a Surface integral. Let $f(x, y, z) = x^2 + y^2 + z^2$. Consider the surface parameterized by

$$\vec{r}(u, v) = \langle u, u + v, 1 \rangle.$$

where the parameters u and v vary as

$$0 \leq u \leq 1$$
$$0 \leq v \leq 1$$

Set up an integral computing the integral of f over the given surface.

$$dS = |\vec{r}_u \times \vec{r}_v|\, du\, dv$$
$$= |\langle 1, 1, 0\rangle \times \langle 0, 1, 0\rangle|\, du\, dv$$
$$= |\langle 0, 0, 1\rangle|\, du\, dv = du\, dv$$
$$\int_0^1\int_0^1 (u^2 + (u+v)^2 + 1^2)\, du\, dv$$

Flux Integral through a surface I. Compute the flux of the vector field $\vec{F} = \langle x, y, z \rangle$ through the surface parameterized by

$$\vec{r}(u, v) = \langle u, u + v, 1 \rangle.$$

where the parameters u and v vary as

$$0 \leq u \leq 1$$
$$0 \leq v \leq 1$$

From before $\vec{r}_u \times \vec{r}_v = \langle 0, 0, 1\rangle$

$$\iint \vec{F} \cdot \hat{n}\, dS = \iint \vec{F} \cdot \vec{r}_u \times \vec{r}_v\, du\, dv$$
$$= \int_0^1\int_0^1 \langle u, u+v, 1\rangle \cdot \langle 0, 0, 1\rangle\, du\, dv$$
$$= \int_0^1\int_0^1 1\, du\, dv = 1$$

Flux Integral through a surface II. Compute the flux of the vector field $\vec{F} = \langle x, y, 0 \rangle$ through the unit sphere.

$\operatorname{div} \vec{F} = 2$. Since the sphere is a closed surface

$$\iint \vec{F} \cdot \vec{n} \, dS = \iiint_{x^2+y^2+z^2 \leq 1} \operatorname{div} \vec{F} \, d\mathrm{vol} = 2 \cdot \mathrm{vol\ sphere}$$

$$= 8/3 \, \pi$$

Bonus Problem: Worth no points! Let $f(x, y, z)$ be a vector field in 3 variables. Compute the quantity

$$\Delta f := \operatorname{div}(\operatorname{grad}(f)).$$

We say that f is *harmonic* if $\Delta f = 0$. Show that whenever f is harmonic, it has no local maxima or minima.