Quiz, Nov. 13

Name:

Setting up a Surface integral. Let $f(x, y, z) = x^2 + y^2 + z^2$. Consider the surface parameterized by

$$\vec{r}(u, v) = \langle u, u + v, 1 \rangle.$$

where the parameters u and v vary as

$$0 \leq u \leq 1$$
$$0 \leq v \leq 1$$

Set up an integral computing the integral of f over the given surface.

Flux Integral through a surface I. Compute the flux of the vector field $\vec{F} = \langle x, y, z \rangle$ through the surface parameterized by

$$\vec{r}(u, v) = \langle u, u + v, 1 \rangle.$$

where the parameters u and v vary as

$$0 \leq u \leq 1$$
$$0 \leq v \leq 1$$

Flux Integral through a surface II. Compute the flux of the vector field $\vec{F} = \langle x, y, 0 \rangle$ through the unit sphere.

Bonus Problem: Worth no points! Let $f(x, y, z)$ be a vector field in 3 variables. Compute the quantity

$$\Delta f := \text{div}(\text{grad}(f)).$$

We say that f is *harmonic* if $\Delta f = 0$. Show that whenever f is harmonic, it has no local maxima or minima.