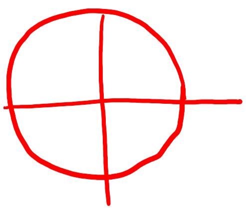
Quiz, Nov. 6

Name:

Line Integral. Compute the line integral of $\langle 0, x\rangle$ over the curve C parameterized by

$$\vec{r}(t) = \langle 3\cos t, 3\sin t\rangle$$

where t goes between 0 and 2π.



$$\text{curl}\, F = \frac{d}{dx} x - \frac{d}{dy} 0 = 1$$

$$\Rightarrow \oint \vec{F}\cdot d\vec{r} = \iint_R \text{curl}\, \vec{F}\, dA = \iint_R 1\, dA = \pi(3)^2$$

$$= 9\pi.$$

Flux Integral, I. Set up, but do not compute, an integral giving the flux of the vector field

$$\langle 4x, 2y^2 + x\rangle$$

through the curve $\langle t^2, t+1\rangle$, where t goes between 0 and 5.

$$\vec{r} = \langle t^2, t+1\rangle$$

$$\vec{r}\,' = \langle 2t, 1\rangle$$

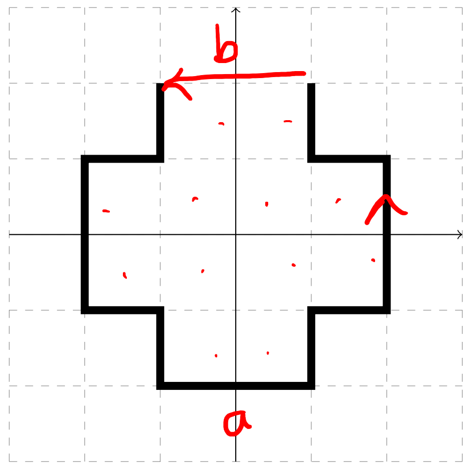
$$\vec{n}\cdot ds = \langle 1, -2t\rangle dt.$$

$$\int \vec{F}\cdot\hat{n}\, ds =$$

$$\int \langle 4t^2, 2(t+1)^2 + t^2\rangle \langle 1, -2t\rangle dt$$

$$= \int 4t - 2t(2(t+1)^2 + t^2)\, dt$$

Flux Integral, II. Give the flux of the vector field $\langle x, 0\rangle$ through the curve drawn below. (The grid drawn is a unit grid.)



$\operatorname{div} F = 1.$

$$\oint_{a \cup b} \vec{F} \cdot \hat{n}\, ds = \iint \operatorname{div} F\, dA = 12$$

$$\oint_{b} \vec{F} \cdot \hat{n}\, ds = \int \langle x, 0\rangle \cdot \langle 0, 1\rangle\, ds = 0$$

$$\Rightarrow \int_{a} \vec{F} \cdot \hat{n}\, ds = 12.$$

Bonus Problem. *Worth no points!* If $\vec{F} = \langle P, Q\rangle$, let the *perpendicular field* for $\vec{F}$ be

$$\vec{F}^{\perp} := \langle Q, -P\rangle$$

Relate the curl of $\vec{F}$ to the divergence of $\vec{F}^{\perp}$, and state why Green's theorem and the divergence theorem are related for these two vector fields.