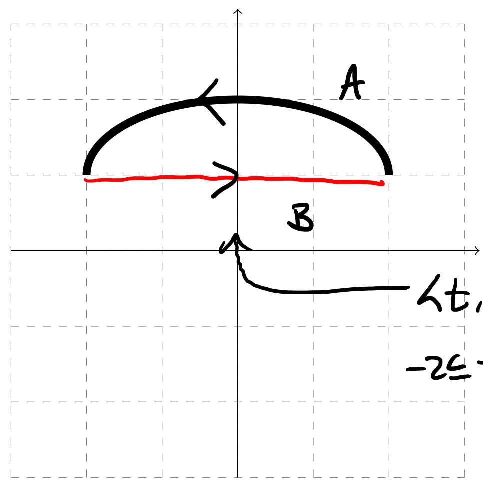
Using Green's theorem. Use Green's theorem to compute the line integral of the vector field $\langle -y, x\rangle$ over the drawn semi-ellipse. (Hint: You can close this up to a closed curve by adding in the line between $(2,1)$ and $(-2,1)$. The area of an ellipse is πab, where a and b are minor and major axis.)



$\langle t, 1\rangle \rightsquigarrow d\vec{r} = \langle 1, 0\rangle dt$

$-2 \le t \le 2$

$\int_A \vec{F}\cdot dr = 2\pi + 4$

Notice that $\quad$ curl $\langle -y, x\rangle = 2.$

$$\oint_C \vec{F}\cdot dr = \iint_A \text{curl}\,\vec{F} = \iint_A 2 = 2\cdot\frac{2\cdot 1\cdot\pi}{2} = 2\pi.$$

$$\|$$

$$\int_A \vec{F}\cdot dr + \int_B \vec{F}\cdot dr = \int_A \vec{F}\cdot dr + \int_{-2}^{2} \langle -1, t\rangle\cdot\langle 1, 0\rangle dt = \int_{-2}^{2} -1\,dt + \int_A \vec{F}\cdot dr$$

$$= -4 + \int_A \vec{F}\cdot dr$$

Computing Flux. Compute the flux of the vector field $\langle x, y\rangle$ through the curve $\vec{r}(t) = \langle \cos t, \sin t\rangle$ where t varies between 0 and $\pi/2$.

$\hat{n} = \langle \cos t, \sin t\rangle$

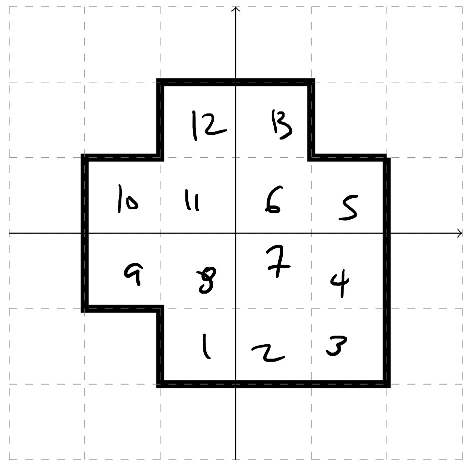
$ds = 1\,dt$

$$\int \vec{F}\cdot\hat{n}\,ds$$

$$\int_0^{\pi/2} \langle \cos t, \sin t\rangle\cdot\langle \cos t, \sin t\rangle\,dt$$

$$= \pi/2.$$

Divergence Theorem. Compute the flux of the vector field $\langle x, y \rangle$ through the curve drawn below.



Divergence Th$^{\underline{m}}$

$\operatorname{div} \vec{F} = 1 + 1 = 2$

$\oint_C \vec{F} \cdot \hat{n}\, ds = \iint_R \operatorname{div} \vec{F} \cdot dA = \iint_R 2 \cdot dA = 26.$