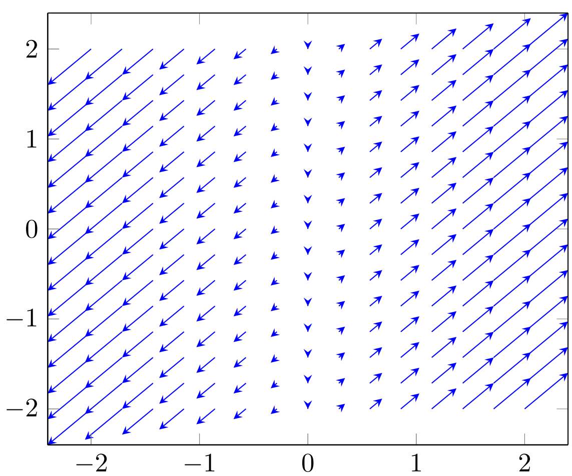
Quiz, Oct. 30

Name:

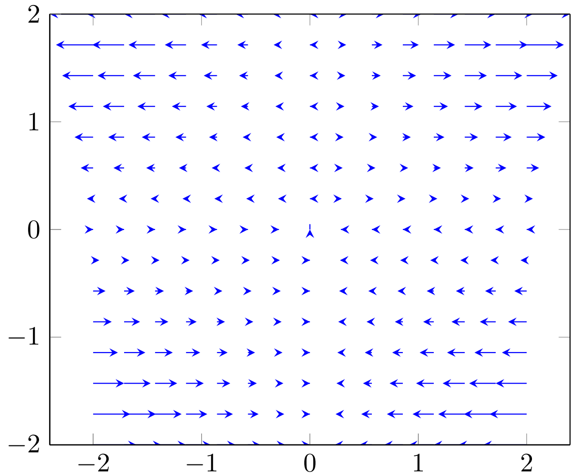
Identifying Vector Fields. Match the following vector fields to their plots.

$\langle x, y\rangle$ D	$\langle xy, 0\rangle$ B
$\langle x, x\rangle$ A	$\langle y, 0\rangle$ C

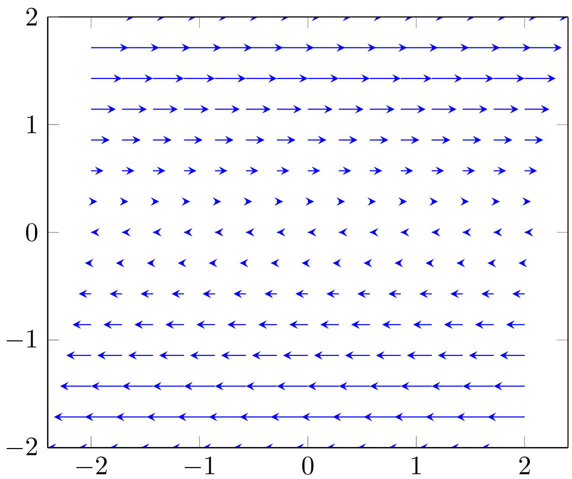
(a)



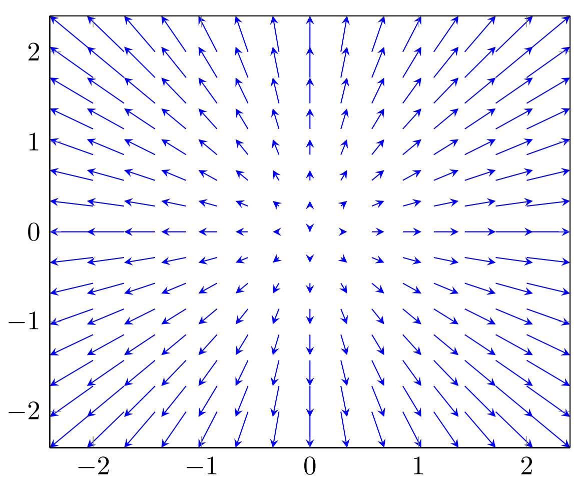
(b)



(c)



(d)



Line Integral of a function. Set up *but do not compute* the integral of the function $f(x, y) = xy$ along the curve C parameterized by

$$x(t) = t$$
$$y(t) = \frac{1}{t}$$

where t goes between 1/2 and 2.

$$ds = \sqrt{1 + \left(\frac{1}{t^2}\right)^2}\, dt$$

$$\int_{\frac{1}{2}}^{2} f(t, \tfrac{1}{t})\, ds = \int_{\frac{1}{2}}^{2} 1 \cdot \sqrt{1 + \frac{1}{t^4}}\, dt.$$

Line Integral of a vector field. Integrate the vector field $\vec{F}(x, y) = \langle x, y \rangle$ along the curve C parameterized by

$$x(t) = t$$
$$y(t) = 2t - 1$$

Where t goes from 0 to 3.

$$dr = \langle 1, 2 \rangle$$

$$\int \langle x, y \rangle \cdot \langle 1, 2 \rangle\, dt$$
$$= \int_0^3 \langle t, 2t - 1 \rangle \cdot \langle 1, 2 \rangle\, dt$$
$$= \int_0^3 t + 4t - 2\, dt = \frac{5}{2}t^2 - 2t = \frac{45}{2} - 6.$$

Bonus Problem. *Worth no points!* Let

$$\vec{F} = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}. \right\rangle$$

Show that $\int_C \vec{F} dr = 0$ if and only if C does not go around the origin.