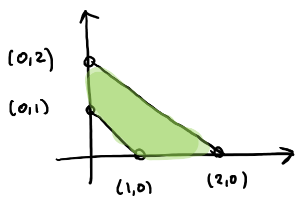
Quiz, Oct. 22

Name:

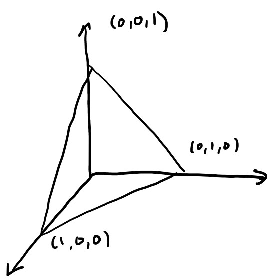
Jacobian (6). Set up, but do not compute, an integral which computes the integral

$$\iint_D (x+y)\,dxdy$$

over the following domain D, using the coordinate change $u = x$ and $v = x + y$.



Triple Integral (7). Compute the integral of the function $f(x, y, z) = x$ over the region in the first octant bounded by $x + y + z \leq 1$.



Triple Integral (7). A cone is given by the region between $0 \le z \le 1 - \sqrt{x^2 + y^2}$. Set up an integral computing the volume of the cone using cylindrical coordinates. Your answer should be in the form of a triple integral. It may help to draw a picture first.

Bonus Problem. *Worth no points!* A function $f(x, y)$ is called *harmonic* if $f_{xx} + f_{yy} = 0$. Show that if a function is harmonic on a compact region R, that the maximum of f must occur on the boundary of the region.

Harmonic functions are time-constant solutions to the heat equation

$$\frac{\partial}{\partial t} f(t, x, y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f(t, x, y)$$

which describes how a function $f(t, x, y)$ describing the tempurature at a point (x, y) at time t behaves. Why does it make physical sense that if we have a time-constant solution to this equation, the maximum must be achieved at the boundary of the region?