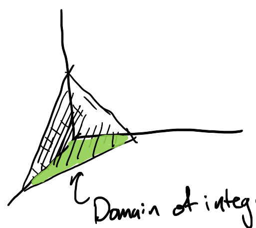
Quiz, Oct. 16

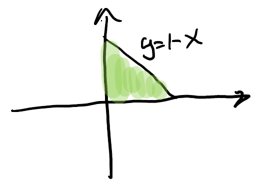
Name:

**Volume (8 Pts).** Compute the volume of the solid which lies in the first octant and is bounded by

$$x + y + z \leq 1$$

(It may be easiest to first draw a picture!)




Volume under graph of

$z = 1-x-y$

$$\int_{x=0}^{x=1}\int_{y=0}^{y=1-x} 1-x-y \quad dy\,dx$$

$$= \int_{x=0}^{x=1} y - xy - \frac{1}{2}y^2 \Big|_0^{1-x} dx = \int_{x=0}^{x=1} 1-x - x(1-x) - \frac{1}{2}(1-x)^2 dx$$

$$= \int_{x=0}^{1} \frac{1}{2}x^2 dx = \frac{1}{2\cdot 3}x^3\Big|_0^1 = \frac{1}{6}.$$

**Rotational Inertia (4 pts).** Compute the rotational inertia of a disk centered at the origin with radius 1, whose density given by $\rho(x,y) = \frac{1}{x^2+y^2}$. $= \frac{1}{r^2}$

$$\text{Inertia} = \iint_D r^2 \rho \quad dA$$

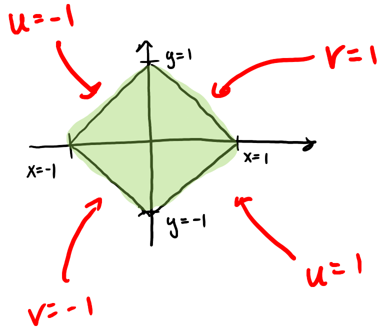
$$= \iint_D r^2 \frac{1}{r^2} dA = \iint_D dA = \text{Area of the domain.}$$

Since the domain is a ○,

$$= \pi r^2 = \pi.$$

**Jacobians (8 Pts).** Compute the integral of $f(x, y) = x + y$ on the drawn area using the change of coordinates

$$u = x + y \qquad\qquad v = x - y$$




$$\left.\begin{array}{l} x = \frac{u+v}{2} \\ y = \frac{u-v}{2} \end{array}\right\} \left| \begin{matrix} x_u & y_u \\ x_v & y_v \end{matrix} \right| = \left| \begin{matrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{matrix} \right|$$

$$= \left| -\frac{1}{4} - \frac{1}{4} \right|$$

Compute Jacobian $\quad = \frac{1}{2}$

$$\iint f \, dA = \int_{v=-1}^{v=1} \int_{u=-1}^{u=1} u \cdot \frac{1}{2} \, du dv$$

$$= \int_{v=-1}^{v=1} u^2 \Big|_{-1}^{1} \cdot \frac{1}{2} du dv = \int_{v=-1}^{v=1} 4 \cdot \frac{1}{2} du dv$$

$$= 2v \Big|_{-1}^{1} = 0.$$

**Bonus Problem.** *Worth no points!* Prove, using calculus, the following geometric identity: