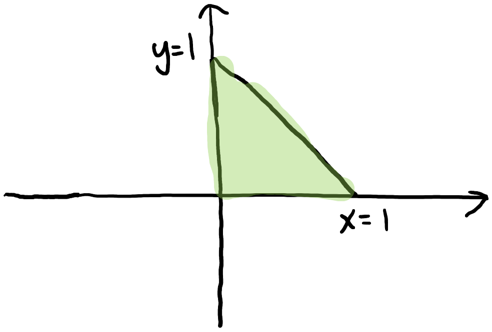
Makeup!

Quiz, ~~Sep 25~~

Name:

Maximizing a function. (8 Pts) Find the place where the function $f(x, y) = x^2 + y^2$ is maximized on the region drawn below



Need to check for critical points

$\nabla f = \langle 2x, 2y \rangle$

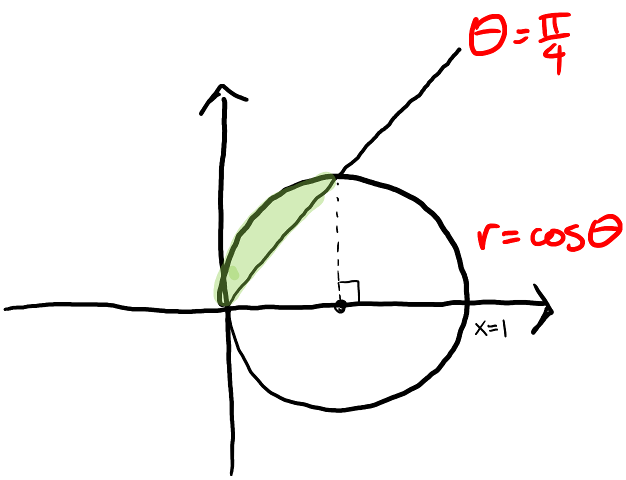
$= 0$ when $x=0, y=0$. Clearly a minimum.

Need to check boundaries

$x=0$	$y=0$	$y=1-x$
$f(0,y)=y^2$	$f(x,0)=x^2$	$f(x,1-x) = x^2 + (1-x)^2 = 2x^2 - 2x + 1$
max at $y=1$	max at $x=1$	$\frac{d}{dx} f(x,1-x) = 4x-2$ critical @ $x=\frac{1}{2}$, $\Rightarrow x^2+y^2 = \frac{1}{2}$.

Maximum is 1.

Polar Coordinates. (4 Pts) Write an integral computes the area of the shaded region using polar coordinates.

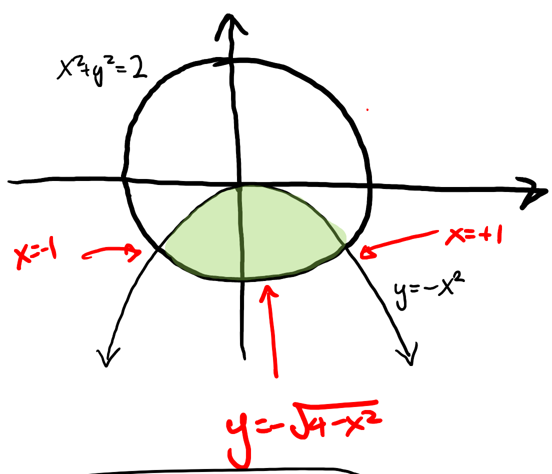


$$= \int_{\theta=0}^{\theta=\pi/4} \int_{r=0}^{r=\cos\theta} r \, dr \, d\theta$$

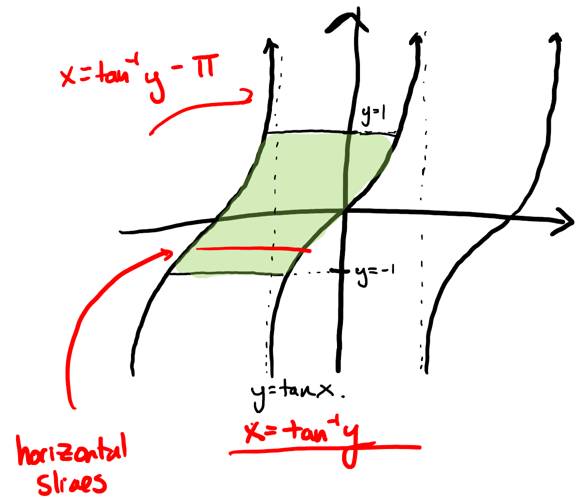
or

$$= \int_0^{\pi/4} \frac{1}{2} \cos^2\theta \, dr \, d\theta$$

Double Integrals. (8 Pts) Write double integrals which compute the areas of each of the shaded regions:



$$\boxed{\int_{x=-1}^{x=+1}\int_{y=-\sqrt{1-x^2}}^{y=-x^2} 1\,dx\,dy}$$



$$\int_{y=-1}^{1}\int_{x=\tan^{-1}y-\pi}^{x=\tan^{-1}y} 1\,dx\,dy$$

Bonus Problem. *Worth no points!* Consider the function $f(r:\theta) = \overset{r\sin(3\theta)}{\cancel{r\cos(3\theta)}}$. Without converting this function into (x, y) components, show that the directional derivatives

$$D_{\langle \cos\theta, \sin\theta\rangle} f(r:\theta)$$

are well defined at the origin. Then show that

$$\nabla f \cdot \langle \cos\theta, \sin\theta\rangle \neq D_{\langle \cos\theta, \sin\theta\rangle}$$

and conclude that this function is not differentiable.