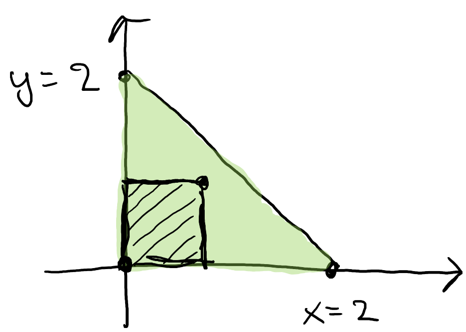
Oct 9

Quiz, ~~Sep 25~~

Name:

Maximizing Area. (8 Pts) Find the maximal area of a rectangle which

- Has sides parallel to the x and y axis
- Has one corner at the origin
- Has opposite corner contained in the shaded region drawn below.



Area of Rectangle is xy.

Want to maximize $f(x,y)=xy$ on region $x\geq 0$, $y\geq 0$, $x+y\leq 2$.

$\nabla f = (y,x)$ has critical point @ $(0,0)$ which is not on interior of region

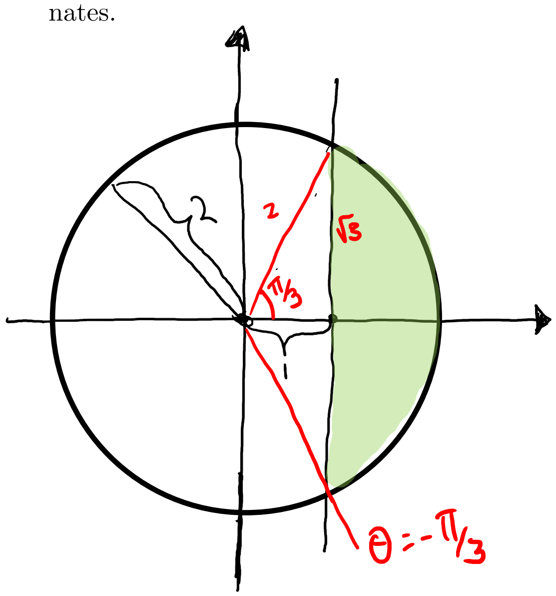
$\Rightarrow f$ is maximized on boundary

Boundary $x=0$	Boundary $y=0$	Boundary $x=t$, $y=2-t$
$f(x,0)=0$	$f(0,y)=0$	$f(t,2-t)=2t-t^2$ $\frac{d}{dt}f = 2-2t=0 \Rightarrow t=1$

Critical point at $(1,1)$ where $f(1,1)=1$.

At corners, $f=0$ $\Rightarrow$ maximum is 1

Polar Coordinates. (4 Pts) Write an integral computes the area of the shaded region using polar coordinates.



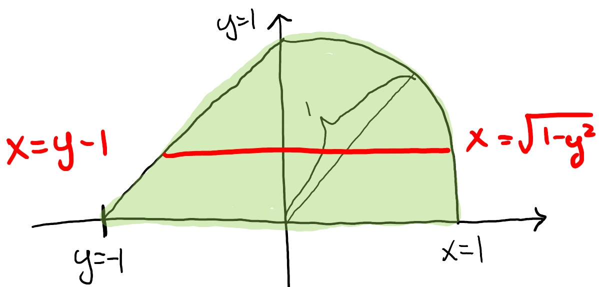
Outer bound is $r_{out}=2$

inner bound is $r_{in}=\sec\theta$

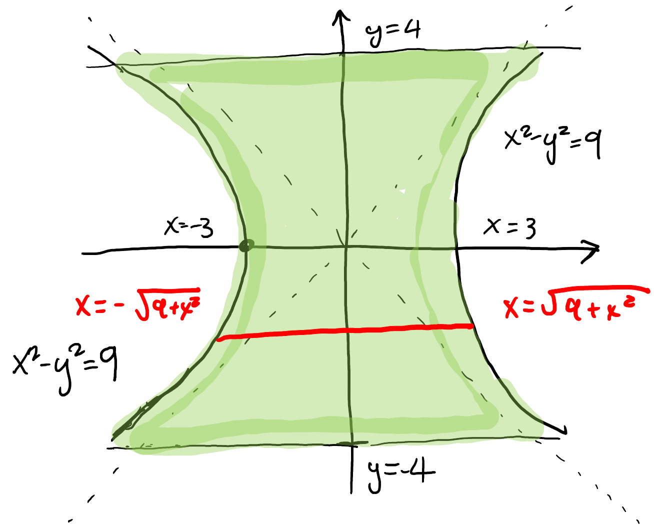
$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}\left(r_{out}^2 - r_{in}^2\right)\,d\theta$$

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3} 2^2-\sec^2\theta \; d\theta.$$

Double Integrals. (8 Pts) Write double integrals which compute the areas of each of the shaded regions:



$$\boxed{\int_{y=0}^{y=1} \int_{x=y-1}^{\sqrt{1-y^2}} 1 \; dx\,dy}$$



$$\int_{y=-4}^{4} \int_{x=-\sqrt{y^2+9}}^{\sqrt{y^2+9}} 1 \; dx\,dy$$

Bonus Problem. *Worth no points!* Can you find a set of numbers a_{ij} where $i, j \in \mathbb{N}$, so that

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = 1$$

but

$$\sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij} = 0.$$

Use these functions to describe a function $f(x, y)$ so that $\iint f dxdy \neq \iint f dydx$. (This will be in an improper integral.)