1. Polar Coordinates

(1) The Limacon is graphed by the polar equation

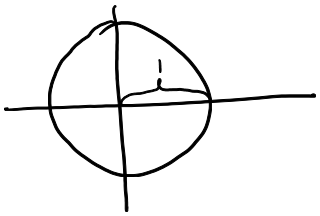
$$r = 1 + c\sin\theta$$

- What shape is this graph when $c = 0$?

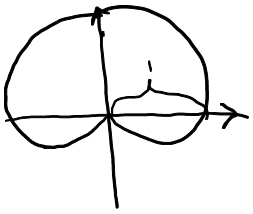
When $c=0$ our equation is

$r=1$

giving us a circle



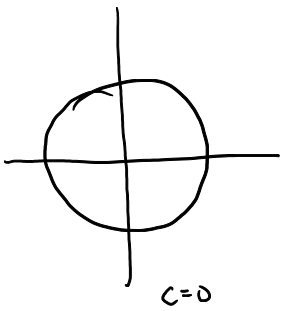
- Sketch a graph of this when $c = 1$.

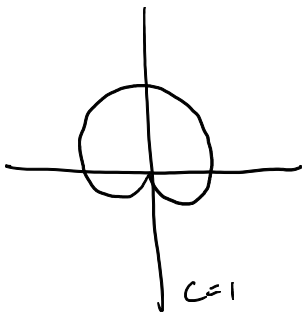


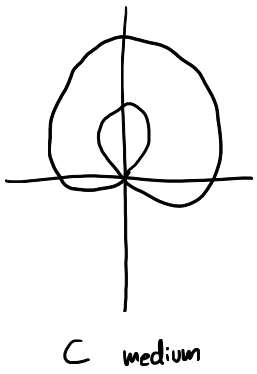
θ	$r = 1+\sin\theta$
0	1
$\pi/2$	2
π	1
$3\pi/2$	0

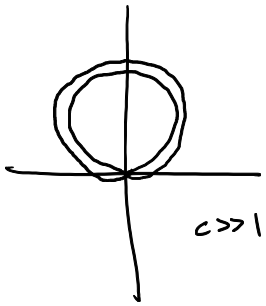
- What is does the graph of $r = \cos(\theta)$ look like?

- Describe how the Limacon changes as c goes to infinity.









(2) Find the intersection points of the following curves:

$$r_1 = 2\cos\theta$$
$$r_2 = 2\sin\theta$$

$2\cos\theta = 2\sin\theta$

$\Rightarrow \theta = \pi/4$

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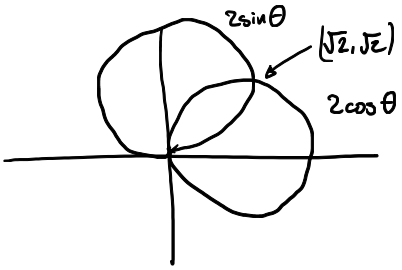
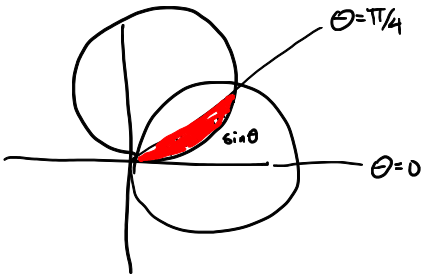
or! whenever $r = 0$.

- Graph these two polar equations to verify your solution.



- Find the area of the region enclosed in both curves.



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\text{Area} = \int_{\theta_1}^{\theta_2} \tfrac{1}{2} r^2 \, d\theta$$

$$= \int_0^{\pi/4} 4\sin^2\theta \, d\theta$$

$$= 2\int_0^{\pi/4} \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= 2\left(\tfrac{1}{2}\theta \Big|_0^{\pi/4} - \frac{\sin 2\theta}{4} \Big|_0^{\pi/4} \right)$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

Total Area is twice the red region

$$= \frac{\pi}{2} - 1$$