Practice Midterm

Name:

Lines and Planes.

(a) The line $\ell(t)$ which is perpendicular to the z axis and goes through the point $(0, 1, 1)$.
(b) Find the equation of a plane containing both the z axis and the point $(0, 1, 1)$.
(c) Show that the line $\ell(t)$ is contained within the plane.

Parameteric Curves. A boy starts at the origin at time $t = 0$ with a velocity of $\langle 1, 1\rangle$. As walks, he spins a yo-yo. From time -2 to time 2, the altitude of the yo-yo can be described by $t^2 + 1$.

(a) Give a parameterization for the position of the yoyo for times $-2 \leq t \leq 2$.
(b) Where does the yoyo maximize its speed?
(c) Compute the distance that the yoyo travels between $t = -2$ and $t = 2$.

Tangent Planes, Min Max. Consider the function $f(x, y) = x^2 - 2xy + y^2 + 3$.

(a) Find the tangent plane to the graph of this function $(1, 1, 3)$.
(b) What critical points does $f(x, y)$ have, and what type are they?
(c) Maximize the function $f(x, y)$ on the region $x^2 + y^2 \leq 4$.

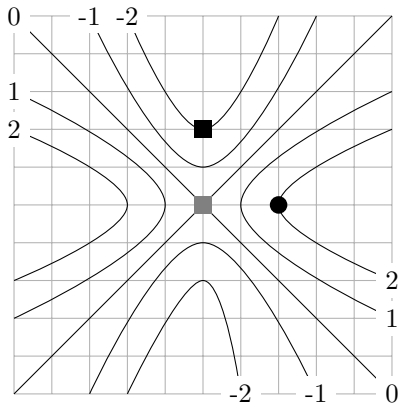
Computations. Let $f(x, y) = x^2 + y^2$. Suppose that we know that $\vec{r}(t) = \langle x(t), y(t)\rangle$ has derivatives

$$|\vec{r}(0)| = 0$$
$$|\vec{r'}(0)| = 1$$
$$|\vec{r''}(0)| = 0$$

Compute

$$\frac{d^2}{dt^2} f(x(t), y(t)).$$

Contour Plots. Estimate the Gradient at each of the points marked with a square. Estimate the equation of the tangent plane at the point $(2, 0, 2)$.



Ants! An ant travels in xy coordinates along the path $(3t, t^2)$ from time 0 to 2. It walks along a hill given whose altitude is given by $f(x, y)$. At time $t = 1$, the ant notices that their rate of altitude change is

$$\frac{d}{dt} f(x(t), y(t))\bigg|_{t=1} = 1$$

and that their altitude is 3.

(a) Estimate the value of f at the point $(6, 3)$.
(b) Suppose further that the magnitude of the gradient $|\nabla f|_{(3,1)}$ is 2. Find the gradient $\nabla f(3, 1)$.