

1. EVEN MORE DERIVATIVES

1.1. **Using the Chain Rule, I.** Suppose that f has a minimum at $(0, 0)$. Let $\vec{r}(t) = \langle at, bt \rangle$ be a line traveling through the origin in the direction of $\langle a, b \rangle$. Show using the chain rule that

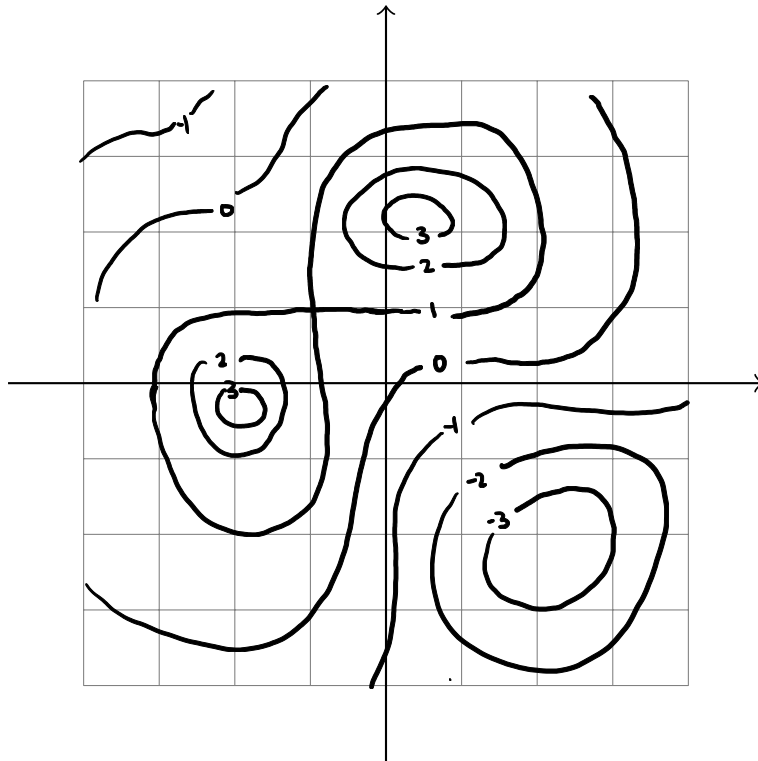
$$\begin{aligned} \left. \frac{d}{dt} f(\vec{r}(t)) \right|_{t=0} &= 0 \\ \left. \frac{d^2}{dt^2} f(\vec{r}(t)) \right|_{t=0} &> 0 \end{aligned}$$

1.2. **Directional Derivatives.** Compute the directional derivative

$$D_{\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle} (x^2 + y^2) \Big|_{(0,1)}$$

1.3. **Finding Maxima and Minima.** Find the point on the plane $z = 2x + y + 1$ which minimizes the distance to $(1, 0, 2)$. Check that this matches the result from using the distance formula from the plane.

1.4. Gradient Vector.



Estimate the gradient vector at $(-1, -3)$, then use this estimate to compute the directional derivative in the $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ direction at that point.