1. Gradients and Chain Rule

1.1. Gradient Calculation. Jack and Jill walk up the hill $f(x, y)$, to fetch a pail of water. The path Jack takes is

$$r(t) = (3t, 2t)$$

and the path that Jill takes is

$$s(t) = (-2t, 3t)$$

Jack reports that at time 0,

$$\frac{d}{dt}(f(3t, 2t))\bigg|_{t=0} = 1$$

and Jill reports that

$$\frac{d}{dt}(f(-2t, 3t))\bigg|_{t=0} = 2$$

- What is the gradient vector ∇f at the origin.
- Suppose additionally that $f(0, 0) = 2$. What is the tangent plane to the graph of f at $(0, 0, 2)$?

$$\frac{d}{dt} f(3t, 2t) = \frac{\partial f}{\partial x} \cdot \frac{d}{dt}(3t) + \frac{\partial f}{\partial y} \frac{d}{dt}(2t)$$

$$3f_x + 2f_y = 1$$

Similarly, 2nd eqn tells us $-2f_x + 3f_y = 2$

system of linear eqn, solve for f_x & f_y.

1.2. Chain Rule I. Use the chain rule, and the function $m(x, y) = xy$ to show the product rule in single variable calculus:

$$(fg)' = f'g + g'f.$$

1.3. **Using the Chain Rule, II.** Suppose we are told that

$$(\nabla f)|_{(x,y)=(1,1)} = \langle 2, 3 \rangle$$

Compute

$$\frac{d}{dt} f(\underbrace{t^2+1}_{x(t)}, \underbrace{t^2+1}_{y(t)})\bigg|_{t=0}$$

$$\frac{d}{dt} f(t^2+1, t^2+1)\bigg|_{t=0} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= 2 \cdot 0 + 3 \cdot 0 = 0.$$