1. Gradients and Chain Rule

1.1. **Gradient Calculation.** Jack and Jill walk up the hill $f(x, y)$, to fetch a pail of water. The path Jack takes is

$$r(t) = (3t, 2t)$$

and the path that Jill takes is

$$s(t) = (-2t, 3t)$$

Jack reports that at time 0,

$$\frac{d}{dt}(f(3t, 2t))\Big|_{t=0} = 1$$

and Jill reports that

$$\frac{d}{dt}(f(-2t, 3t))\Big|_{t=0} = 2$$

- What is the gradient vector ∇f at the origin.
- Suppose additionally that $f(0, 0) = 2$. What is the tangent plane to the graph of f at $(0, 0, 2)$?

1.2. **Chain Rule I.** Use the chain rule, and the function $m(x, y) = xy$ to show the product rule in single variable calculus:

$$(fg)' = f'g + g'f.$$

1.3. **Using the Chain Rule, II.** Suppose we are told that

$$(\nabla f)|_{(x,y)=(1,1)} = \langle 2, 3 \rangle$$

Compute

$$\frac{d}{dt} f(t^2 + 1, t^2 + 1)\Big|_{t=0}$$