Quiz, Sep 18

Name:

Continuity (5pts). Show that the function $\frac{x+y}{\sqrt{x^2+y^2}}$ is not continuous at the origin. (One way to do this is to take limits along $x = y$ and $x = -y$.)

Two methods:

- First, let's look at $f(t,t)$. This is $\frac{t+t}{\sqrt{t^2+t^2}} = \frac{2t^2}{\sqrt{2t^2}}$.

  If t is positive, this is $+1$ so $\lim_{t\to 0^+} f(t) = +\sqrt{2}$
  If t is negative, this is -1. so $\lim_{t\to 0^-} f(t) = -\sqrt{2}$
  $\Rightarrow$ not continuous.

- If we compare this to $f(t,-t)$ we have $\lim_{t\to 0} f(t,-t) = \lim_{t\to 0} \frac{t-t}{\sqrt{2t^2}} = 0$.

  So, this is again shown not to be equal to $\lim_{t\to 0} f(t,t)$.

Tangent Plane (8 pts). Find the equation for the tangent plane to $f(x, y) = xy$ at the point $(1, 1, 1)$.

$\frac{\partial f}{\partial x} = y \qquad \left.\frac{\partial f}{\partial x}\right|_{(1,1)} = 1$

$\frac{\partial f}{\partial y} = x \qquad \left.\frac{\partial f}{\partial y}\right|_{(1,1)} = 1$

Normal of tangent plane is $\langle f_x, f_y, -1\rangle$

Normal of tangent plane at $(1,1,1) = \langle f_x|_{(1,1)}, f_y|_{(1,1)}, -1\rangle$

$= \langle 1, 1, -1\rangle$

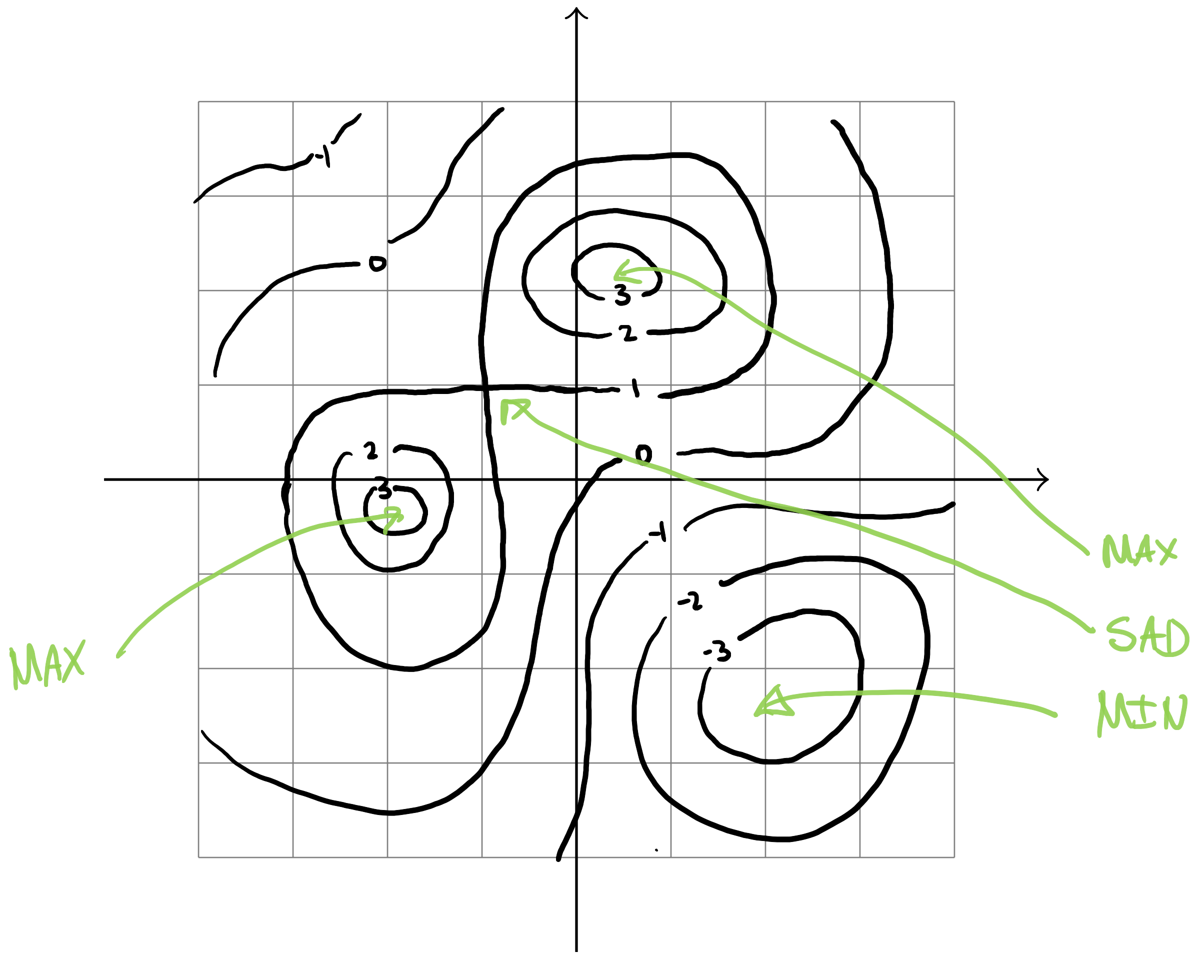
Point on plane $= \langle 1,1,1\rangle$.

Eqn of plane is $\vec{n}\cdot\langle \vec{v} - p_0\rangle = 0 \leadsto$

$\langle 1, 1, -1\rangle \cdot \left(\langle x, y, z\rangle - (1,1,1)\right) = 0$

$x + y - z - 1 = 0$.

Contour Plots. Consider the function of 2 variables whose contour graph is drawn below.



- **4 Points:** Mark all local maximum, minima, or saddle points of f on the contour plot.
- **3 Points:** Write the equation of the tangent plane at the point $(-1, 1, 1)$.

The point $(-1,1)$ is a saddle point

$\Rightarrow$ tangent plane is $\parallel$ to xy plane

$\Rightarrow$ tangent plane is $z = 1$.

Bonus Problem. *Worth no points!* Consider the piecewise defined function

$$f(x,y) = \begin{cases} 0 & \text{Whenever } x \neq y^2 \text{ or } x = y = 0 \\ 1 & \text{Whenever } x = y^2 \text{ and } x \neq y \neq 0. \end{cases}$$

Show that this function is continuous along every line approaching the origin. Also show that this funciton is not continuous.

For any line $l(t) = (x(t), y(t))$, there is constant c small enough so that $t < c$ implies $f(l(t)) = 0$.
This tells us that we are continuous along that line.