1. More On Derivatives

1.1. **Using the Chain Rule, I.** Let $s(t) = f(x(t), y(t))$, and assume that

$$\left.\frac{ds}{dt}\right|_{t=3} = 5$$

Additionally, suppose that you are told that

$$\left.\frac{\partial f}{\partial x}\right|_{(0,2)} = 1 \qquad \left.\frac{\partial f}{\partial y}\right|_{(0,2)} = 1$$

$$\left.\frac{dx}{dt}\right|_{t=3} = 3$$

$$y(3) = 0 \qquad x(3) = 2$$

What is $y'(3)$?

1.2. **Chain Rule in Proofs.** The function $f(x, y, z) = x^2 + y^2 + z^2$ measures the distance squared of a point from the origin. Show that if $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$ with $\vec{r}(0) = \langle 1, 0, 0\rangle$, and $\vec{r}(t) \cdot \vec{r}'(t) = 0$, that $f(x(t), y(t)) = 1$ for all t.
Conclude that the curve $\vec{r}(t)$ is contained in the unit sphere.

1.3. **Implicit Differentiation.** Let $f(x,y) = x^3 + y^2$. Suppose that we have a curve $\vec{r}(t) = \langle x(t), y(t) \rangle$ which traces out a level set of f so that

$$f(x(t), y(t)) = 2.$$

Additionally, suppose that

$$\langle x(0), y(0) \rangle = \langle 1, 1 \rangle.$$

Finally, assume that $x'(0) = 1$. Find the velocity vector $\vec{v}(0)$.

Notice that the velocity vector of $\vec{r}(t)$ is tangent to the 2-level curve at the point $(1,1)$. Use this information to find a tangent line to level set at $(1,1)$.