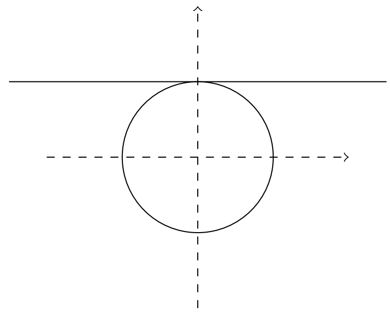
Quiz, Sep 11

Name:

**Intersecting Curves.** Find the points of intersection, if any, between the parametric curves $\vec{r}(t) = \langle t, 1\rangle$ and $\vec{s}(t) = \langle \cos t, \sin t\rangle$.

**Solution:**It helps in this case to draw a picture: the first curve parameterizes a horizontal line which meets the $y$ axis at $y = 1$. The second curve is the unit circle.



Our expectation should be that they intersect at the point $(0, 1)$.
Formally, we can solve this by setting the two equations equal to eachother.

$$\langle t_0, 1\rangle = \langle \cos t_1, \sin t_1\rangle$$

Notice here that I'm using a different variable for the first parameter versus the second parameter (if we use the same parameters, then we will be finding the points where the curves parameterized by these function intersect.) Separating this into two components, the equation on the $y$ coordinates tells me that

$$\sin t_1 = 1$$

which tells me that it must be the case that $t_1 = \pi/2$. From setting the $x$ coordinates equal to eachother I get

$$t_0 = \cos t_1 = \cos(\pi/2) = 0.$$

I can solve this equation, and as a result I get that the points intersect at $(1, 0)$.

**Equation of a plane.** Find the equation of the plane which passes through the origin and contains the line $\langle t+1, t-1, 1\rangle$.

**Solution:**I think the easiest way to solve this is to find three points on the plane, and use this to find the equation of a plane. In this case, we will take the points

$$\begin{aligned}\vec{v}_0 =&\langle 0,0,0\rangle\\ \vec{v}_1 =&\langle 1,-1,1\rangle\\ \vec{v}_2 =&\langle 0,-2,1\rangle\end{aligned}$$

The points are computed by letting $t=0$ and $t=-1$ in the equation of the line. From this information, we can compute two vectors which are parallel to the plane: $\vec{v}_1 - \vec{v}_0$ and $\vec{v}_2 - \vec{v}_0$. Since $\vec{v}_0 = \langle 0,0,0\rangle$, we can just use $\vec{v}_1$ and $\vec{v}_2$ for two vectors which are parallel to the plane.
To finish up, we need to compute a normal direction to the plane. This is given by $\vec{v}_1 \times \vec{v}_2 = \langle 1,-1,-2\rangle$. So, we know that our plane is of the form

$$x - y - 2z + d = 0,$$

and by plugging in the point $\langle 0,0,0\rangle$ into the equation of the plane, we get

$$x - y - 2z = 0.$$

**Parametric curves and lengths.**

- A clock has a second hand which is 4cm long, and a bug travels from the center of the clock to the edge of the clock by walking along the second hand. While the bug travels, the second hand keeps turning. It takes the bug 2 minutes to travel from the center to the end of the second hand, and the bug begins its journey at midnight. Write down a parametric equation for the path of the bug. (Assume that the clock is centered at the origin, that $t = 0$ is midnight, the time $t$ is measured in seconds, and that the 12 o'clock point is oriented along the $y$-axis.)

  **Solution:**It helps to draw a picture. The snail has a radial velocity of
  $$\frac{4\text{cm}}{120s} = \frac{1}{30}\text{cm/s}$$
  and an angular velocity of
  $$\frac{2\pi\text{rad}}{60\text{s}} = \frac{\pi}{30}\text{rad/s}$$
  so we can write out the position of the bug as
  $$\vec{r}(t) = \left\langle \frac{t\sin(\frac{\pi}{30})}{30}, \frac{t\sin(\frac{\pi}{30})}{30} \right\rangle = \frac{1}{30}\left\langle t\sin\left(\frac{\pi t}{30}\right), t\cos\left(\frac{\pi t}{30}\right)\right\rangle$$

- Write down an integral which computes the distance that the bug has traveled (don't evaluate the integral, but simplify it.)
  **Solution:**The velocity of this is
  $$\vec{v}(t) = \frac{1}{30}\left\langle \sin\left(\frac{\pi t}{30}\right) + \frac{\pi t}{30}\sin\left(\frac{\pi t}{30}\right), \cos\left(\frac{\pi t}{30}\right) - \frac{\pi t}{30}\cos\left(\frac{\pi t}{30}\right)\right\rangle$$
  Which gives us a speed of
  $$|\vec{v}(t)| = \frac{1}{30}\sqrt{1 + \left(\frac{\pi t}{30}\right)^2}$$
  so our length becomes
  $$\int_0^{120}\left(\frac{1}{30}\sqrt{1+\left(\frac{\pi t}{30}\right)^2}\right)dt$$

**Bonus Problem.** *Worth no points!* Show that if a curve $\vec{r}(t)$ has constant non-zero distance from a plane, that it is necessarily the case that it is parallel to the plane.