

WORKSHEET, SEP 8

- (1) A wooden stick is 10 meters long and 40 centimeters in diameter. A string wound around the stick in a spiral. The spiral winds around the stick 20 times (so the distance between each winding of the string is .5 meters.) How much string is used in the spiral?

- (2) Let $\langle x(t), y(t) \rangle$ be a parametric curve. Let L be the length of the curve from $t = 0$ to $t = 1$. Make a geometric argument for why

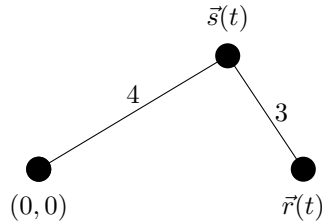
$$L \geq |x(1) - x(0)|,$$

and prove this using the arc-length formula. When does this equality hold? (Draw pictures!)

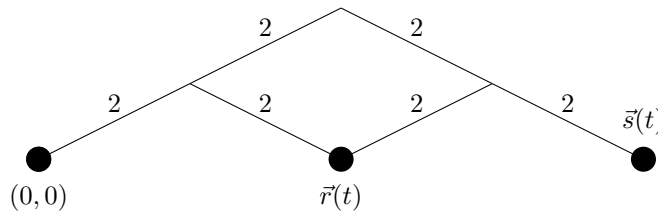
- (3) For which values of t is the curve given by $\langle 1 - t^3, 1 - t^2, 1 + t \rangle$ parallel, skew, intersecting or tangent to the z axis?

A linkage is a contraption made from steel bars and hinges. The steel bars are not allowed to change length. A single hinge is fixed at the point $(0, 0)$, but all the other hinges are allowed to rotate freely and move. A single hinge is called the “input point”, while another hinge is called the “output point.” By pushing the location of the input point around with a parametric equation $\vec{r}(t)$, we get a new parametric equation $\vec{s}(t)$ at the output point. For each linkage, figure out the output parametric equation for the corresponding input parametric equation.

- (1) Let $\vec{r}(t) = \langle 5 \cos(t), 5 \sin(t) \rangle$.



- (2) Let $\vec{r}(t) = \langle x(t), y(t) \rangle$. (Note: the line going from the origin labeled 2 and 2 is always straight in this example.) This linkage is historically called a Pantograph.



- (3) Let $\vec{r}(t) = \langle 1 + 2 \cos \theta, \sin \theta \rangle$

