Worksheet, Sep 6

(1) Show that the curve

$$\vec{r}(t) = \langle 1+t^2, 1+t^2, 1+t \rangle$$

does not intersect the plane P

$$-2x + 3y + z - 1 = 0.$$

Solution: Let's define the functions for the components of our curve as $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$. Then what we want to show is that

$$-2x(t) + 3y(t) + z(t) \neq 1$$

for any value of t, as this means that the points on the curve never satisfy the equation of the plane. When we plug this in, we get

$$-2(1+t^2) + 3(1+t^2) + (1+t) - 1 = t^2 + t + 1$$

The discriminant of this polynomial is negative, so it has no zeros. Therefore, it does not intersect the plane.

(2) Compute the function $d(t)$ describes the distance between the plane P and the point $\vec{r}(t)$.
Solution: The equation for determining the distance from a point to the plane is given by taking component to the normal vector (and subtracting off the distance to the origin,)

$$d(t) = \frac{|ax(t) + by(t) + cz(t) + d|}{\sqrt{a^2 + b^2 + c^2}}$$

which, when expanded becomes

$$\frac{|-2(1+t^2) + 3(1+t^2) + (1+t) - 1|}{\sqrt{14}} = \frac{t^2 + t + 1}{\sqrt{14}}$$

(3) Using single variable calculus, find the time t_{min} which minimizes the function $d(t)$. What does this mean about $\vec{r}(t_0)$?
Solution: The function is minimized when $\frac{2t+1}{\sqrt{14}} = 0$, or when $t = \frac{-1}{2}$.

(4) Compute $\vec{n}$, the normal vector to the plane. Compute the velocity vector $\vec{v}(t) = \frac{d\vec{r}}{dt}$.
Solution: $\vec{n} = \langle -2, 3, 1 \rangle$. Differenting in each component gives us

$$\vec{v}(t) = \langle x'(t), y'(t), z'(t) \rangle = \langle 2t, 2t, 1 \rangle$$

(5) Geometrically explain why

$$\vec{n} \cdot \vec{v}(t_0) = 0$$

and verify this identity.
Solution: At the place where $\vec{r}(t)$ is closest to the plane, it's velocity should be parallel to the plane.

$$\langle -2, 3, 1 \rangle \cdot \langle 2t, 2t, 1 \rangle = -4t + 6t + 1 = 2t + 1$$

which is zero when $t = \frac{-1}{2}$.