Quiz, Feb 8

Name:

0.1. **Cross Product.** Compute the following Cross Products:

$$\langle 1,2,1\rangle \times \langle 1,4,5\rangle \qquad\qquad \langle 7,-1,3\rangle \times \langle -14,2,-6\rangle$$

Solution: The first cross product is given by

$$\begin{aligned}\langle 1,2,1\rangle \times \langle 1,4,5\rangle =& \begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \hat{k} \\ =&(10-4)\hat{\imath} - (5-1)\hat{\jmath} + (4-2)\hat{k} \\ =&\langle 6,-4,2\rangle\end{aligned}$$

For the second cross product, notice that the second vector is -2 times the first one. Therefore, the cross product is zero (as the vectors are parallel.)

0.2. **Lines and Planes.** Find the equation of a plane which is parallel to the plane $2x-2y+z=2$ and is distance 6 away.
Solution: Since our new plane is parallel to the old one, we know that the normal vector to each plane is the same. Let's call this vector $\vec{N} = \langle 1,2,1\rangle$. So we know the new plane has equation

$$2x-2y+z=c$$

for some different value c. We just need to find a point which is distance 6 from the old plane and plug it in to find the value of c. Take any point you want on the plane– like (0, 0, 2), then add a normal vector of length two. The corresponding unit normal vector is $\hat{N} = \langle 2/3, -2/3, 1/3\rangle$. So a point distance 6 from the old plane is

$$(0,0,2) + 6\hat{N} = (4,-4,4)$$

Plugging this in, we get

$$2x-2y+z=20$$

0.3. **Projections.** Show that $\text{Comp}_{\vec{u}}(\vec{v}\times\vec{u}) = 0$.
Solution: The geometric explanation is that $\vec{u}$ is perpendicular to $\vec{v}\times\vec{u}$, so the component of one onto the other is going to be zero.
Computationally,

$$\begin{aligned}\text{Comp}_{\vec{u}}(\vec{v}\times\vec{u}) =& \frac{\vec{v}\cdot(\vec{v}\times\vec{u})}{\vec{u}} \\ =&\frac{1}{|\vec{v}|}\langle v_x, v_y, v_z\rangle \cdot \langle v_y u_z - v_z u_y, -(v_x u_z - v_z u_x), v_x u_y - v_y u_x\rangle \\ =&v_x v_y u_z - v_x v_z u_y - v_y v_x u_z + v_y v_z u_x + v_z v_x u_y - v_z v_y u_x \\ =&0\end{aligned}$$

Bonus Problem. *Worth no points!* Let $F_1, \ldots F_k$ describe the faces of a polyhedron. Let $\vec{N}_1, \ldots \vec{N}_k$ be the outward pointing normal vectors to these faces, with $|\vec{N}_i| = \text{Area}(F_i)$. Prove that for any vector $\vec{b}$, we have

$$\vec{b}\cdot\vec{N}_1 + \vec{b}\cdot\vec{N}_2 + \cdots + \vec{b}\cdot\vec{N}_k = 0$$