# Worksheet, Sep 1

## 0.1. **Lines and Planes.**

(1) Let $\ell$ defined by

$$x = t + 1$$
$$y = -1$$
$$z = -t$$

Find a new line $\ell_1$ which Contains the origin and is perpendicular to our original line $\ell$.
**Solution:**The vector from the origin to the point $\ell(t)$ is simply $\langle t+1, -1, -t\rangle$. We want to know when this is perpendicular to the direction that $\ell$ travels in, which is $\langle 1, 0, -1\rangle$. So we want to find the value of $t$ solving

$$0 = \langle t+1, 2t-1, -t\rangle \cdot \langle 1, 0, -1\rangle = t + 1 + 4t - 2+$$

which is when $t = \frac{1}{2}$. So the direction that the line travels in is

$$\ell(1/6) = (3/2, -1, -1/2)$$

We can scale this direction to something easier to look at, so the direction the new line travels in is $\vec{v} = \langle 3, -2, -1\rangle$.

**Solution:**Let $\vec{v} = \langle 1, 0, 1-\rangle$ be the direction that the line travels in. Let $\vec{p}$ be a point on the line. Then $\vec{n} = \vec{v} \times \vec{p}$ the normal to the plane that contains both the line $\ell$ and the origin. Notice now that the line perpendicular to $\ell$ and the origin is contained in this plane, so the direction of this line must be $\vec{n} \times \vec{v}$.

(2) Let $P_1, P_2$ and $P_3$ be three planes. Suppose that $P_1, P_2, P_3$ all contain a common line. Show that the respective normal vectors $\hat{n}_1, \hat{n}_2$, and $\hat{n}_3$ to these planes all lie in the same plane. **Solution:**Let $\vec{v}$ be the direction of the common line. Then $\vec{n}_1 \cdot \vec{v} = \vec{n}_2 \cdot \vec{v} = \vec{n}_3 \cdot \vec{v} = 0$. Therefore, they are contained in the normal plane to $\vec{v}$.

(3) Describe an algorithem which finds the minimal distance between 2 lines (which does not involve taking a derivative!)
**Solution:**Let the first line be given by $t\vec{v}_1 + p_1$, and the second by $t\vec{v}_2 + p_2$. Then the shortest distance between these two lines is a line which is perpendicular to both of them. Let $\vec{n} = \vec{v}_2 \times \vec{v}_3$ be a vector perpendicular to both $\ell_1$ and $\ell_2$. Then take any vector $\vec{w}$ with ends on $\ell_1$ and $\ell_2$, and take the component $\text{comp}_{\vec{n}}\vec{w}$.

0.2. **Parametric Functions.**

(1) Show that the curve

$$\vec{r}(t) = \langle 1 + t^2, 1 + t^2, 1 + t \rangle$$

does not intersect the plane

$$-2x + 3y + z = 1.$$

(2) Find $\vec{r}(t)$, the parametric equation for the circle centered at the origin with radius 1, and $\vec{s}(t)$ the parametric equation for the circle centered at $(1, 0)$. Find the points of intersection (if any) between these two curves.

(3) Two gears of radius 1 are placed side by side, with the first gear centered at the origin. A point $p$ is marked on the boundary of the second gear. What is the parametric equation describing the path of the point as we rotate the second gear around the first gear?